Day 261 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 4th term of an arithmetic sequence is 32 and the 9th term is 72.
Find the first term and common difference, and the sum of the first 30 terms.

$u_9 - u_4 = 5\,d = 72 - 32$ so $d = 8$

$a + 3\,d = 32$ so $a = 8$

$S_{30} = \frac{30}{2}(2 \times 8 + 29 \times 8)$

Answer: a = 8, d = 8, S_{30} = 3720

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A college has 540 students: 200 Arts, 60 Science and 280 Humanities.
A stratified sample of size 27 is taken.
Find the number of students sampled from each group.

Sampling fraction = $\frac{27}{540} = \frac{1}{20}$

Arts: $200 \times \frac{1}{20} = 10$

Science: $60 \times \frac{1}{20} = 3$

Humanities: $280 \times \frac{1}{20} = 14$

Answer: Arts 10, Science 3, Humanities 14

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^3 (2x)^2\,dx$.

Integrate: $[\frac{(2x)^3}{6}]$

Substitute the limits and subtract: 36

Answer: 36

4 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (2 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $6g - T = 6a$; B: $T - 2g = 2a$
$a = 4g \div 8 = 4.9$, so $T = 2(g + a) = 29.4$
When A lands, B has risen 2 m with $v^2 = 2 \times 4.9 \times 2 = 19.6$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = 1$
Total = 2 + 1
Answer: (a) 29.4 N (b) 3 m

5 Asymptotes after transformation (All boards)

The curve y = f(x), where $f(x) = \frac{4}{x+2} - 2$, is translated by $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): $x = -2$, $y = -2$

Translate by $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$: $x = -5$, $y = -5$

New curve: $y = \frac{4}{x+5} - 5$

$y = 0$: $\frac{4}{x+5} = 5$, so $x + 5 = \frac{4}{5}$

Answer: $x = -5$, $y = -5$; crosses x-axis at $(-\frac{21}{5}, 0)$

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6 Stationary points with trigonometric and exponential terms (All boards)

$y = e^{2x} - 12e^x + 10x$.

Show that the stationary points occur where $e^x = 1$ and $e^x = 5$, and find the exact y-coordinate of each, with its nature.

$\frac{dy}{dx} = 2e^{2x} - 12e^x + 10 = 2(e^x - 1)(e^x - 5)$

So $e^x = 1$ or $e^x = 5$, i.e. $x = \ln 1$ or $x = \ln 5$

$\frac{d^2y}{dx^2} = 4e^{2x} - 12e^x = e^x(4e^x - 12)$ changes sign: negative at the smaller root, positive at the larger

Answer: $x = 0$, $y = -11$ (maximum); $x = \ln 5$, $y = 10 \ln 5 - 35$ (minimum)

7 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−4, 0) and passes through B(−1, 1).

Find the equation of the circle.

Touches the x-axis at A, so the centre is (−4, r) and the radius is r.

B on the circle: $(3)^2 + (1 - r)^2 = r^2$

$9 + 1 - 2r = 0$ so $r = 5$

Answer: $(x + 4)^2 + (y - 5)^2 = 25$

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 + 2xy + 2y^2 = 9$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x + 2y}{2x + 4y}$

Horizontal tangent: $2x + 2y = 0$, so $y = -x$

Substitute: $x^2 = 9$, so $x^2 = 9$ and $x = \pm 3$

Points: (3, −3) and (−3, 3)

Answer: (3, −3) and (−3, 3)

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (15 m, 30 kg) rests with A on rough ground, $\mu = 0.3$, and B against a smooth wall, B being 12 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(15^2 - 12^2)} = 9$ m from the wall

Vertical: $R = (30 + 50)g$. Limiting friction: $F = \mu R = 0.3(30 + 50)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 9/15$):

$S \times 12 = 30g \times 4.5 + 50g \times (9x \div 15)$

$24 \times 12 = 135 + 30x$ (g cancels), so $x = 5.1$

Answer: 5.1 m

10 Hypothesis test for a Poisson mean (CIE)

Accidents occur as a Poisson process at a mean rate of 5 per week.

After a change, the number recorded in one week is 13.

Test at the 5% level whether the mean rate has increased.

$H_0: \lambda = 5$, $H_1: \lambda > 5$; $X \sim Po(5)$

$P(X \geq 13) = 1 - P(X \leq 12) = 0.00202$

$0.00202 < 0.05$

Reject H_0: evidence the rate has increased.

Answer: p-value 0.00202; reject H_0