Day 260 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic modelling (All boards)

A charity raises £600 in its first year and in each later year raises £30 more than in the previous year.
(a) How much does it raise in year 11? (b) In which year does the total raised first exceed £5000?

(a) $u_{11} = 600 + 10 \times 30 = 900$

(b) $S_m = \frac{m}{2}(1200 + (m - 1) \times 30) > 5000$

$S_7 = 4830 < 5000$ and $S_8 = 5640 > 5000$

Answer: (a) £900 (b) year 8

2 Venn diagrams and conditional probability (All boards)

P(A) = 0.25, P(B) = 0.6 and P(A ∩ B) = 0.05.
Find
(a) P(A ∪ B)
(b) P(A | B)
(c) P(A′ ∩ B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.6 - 0.05 = 0.8$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.6} = \frac{1}{12}$

(c) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

Answer: (a) 0.8 (b) $\frac{1}{12}$ (c) 0.2

3 Differentiating sec, cosec, tan and cot (All boards)

y = x sec(2x).

Find the exact gradient at $x = \frac{\pi}{6}$.

Product rule: $\frac{dy}{dx} = \sec(2x) + x(2\sec(2x)\tan(2x))$

At $x = \frac{\pi}{6}$: $\sec(2x) = 2$ and the derivative term $= 4\sqrt{3}$

Gradient $= 2 + \frac{\pi}{6} \times 4\sqrt{3} = \frac{2\sqrt{3}\pi + 6}{3}$

Answer: gradient $\frac{2\sqrt{3}\pi + 6}{3}$

4 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = −1
Line AB: y − 1 = 0(x − −3). Solve to get D = (−1, 1)
Area = ½ × AB × CD = 16
Answer: (a) x = −1 (b) D(−1, 1), area 16

5 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{7}{24}$. It moves 10.78 m in 5 s.
Find (a) the coefficient of friction, (b) the speed after 5 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $10.78 = \frac{1}{2} \times a \times 5^2$, so $a = 0.8624$
Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$
$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.28 - 0.96\mu) = 0.8624$
$0.28 - 0.96\mu = 0.088$, so $\mu = 0.2$
$v = at = 0.8624 \times 5$
Answer: $\mu = 0.2$; 4.312 m s^{-1}

6 Hypothesis test for a normal mean (All boards)

The mean time taken (minutes) is normally distributed with standard deviation 10; it was previously 44.
A random sample of 36 has mean 42.1.
Test, at the 5% level, whether the mean has changed.

H_0: $\mu = 44$, H_1: $\mu \neq 44$

$\bar{X} \sim N(44, \frac{10^2}{36})$, $z = \frac{42.1 - 44}{\frac{10}{6}} = -1.14$

p-value = 0.254 (two-tailed) > 0.05
Do not reject H_0: no evidence that the mean has changed.

Answer: $z = -1.14$, $p = 0.254$; do not reject H_0

7 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = 2t + 1$, $y = t^2 - 4t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -\frac{1}{2}$, in the form $ax + by + c = 0$ (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = t - 2$

$t = -\frac{1}{2}$: point $(0, \frac{9}{4})$, gradient $-\frac{5}{2}$

Normal gradient $= -1 \div (-\frac{5}{2}) = \frac{2}{5}$, so normal: $8x - 20y + 45 = 0$

Answer: $\frac{dy}{dx} = t - 2$; $8x - 20y + 45 = 0$

8 Asymptotes after transformation (All boards)

The curve $y = f(x)$, where $f(x) = \frac{3}{x+2} + 2$, is translated by $\begin{pmatrix}1\\1\end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of $y = f(x)$: $x = -2$, $y = 2$

Translate by $\begin{pmatrix}1\\1\end{pmatrix}$: $x = -1$, $y = 3$

New curve: $y = \frac{3}{x+1} + 3$

$y = 0$: $\frac{3}{x+1} = -3$, so $x + 1 = -1$

Answer: $x = -1$, $y = 3$; crosses x-axis at $(-2, 0)$

9 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$; friction $F = \mu R = 0.1 \times 6\, g \cos\alpha$

B: $10g - T = 10a$
A: $T - 6g\sin\alpha - F = 6a$

Adding: $g(10 - \frac{72}{13} - \frac{3}{13}) = 16a$

(a) $a = 2.59$; (b) $T = 10(g - a)$

Answer: (a) 2.59 m s^{-2} (b) 72.1 N

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10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 - xy + 4y^2 = 62$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{4x - y}{-x + 8y}$$

Horizontal tangent: $4x - y = 0$, so $y = 4x$

Substitute: $62x^2 = 62$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, 4)$ and $(-1, -4)$

Answer: $(1, 4)$ and $(-1, -4)$