Day 26 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 - 6t)\mathbf{i} + (4t^2 - 8t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 2, (b) the value of t when P is moving parallel to $\mathbf{i}$.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 - 6)\mathbf{i} + (8t - 8)\mathbf{j}$
(a) t = 2: $\mathbf{v} = 6\mathbf{i} + 8\mathbf{j}$
Speed = $\sqrt{(6^2 + 8^2)} = \sqrt{100}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $8t - 8 = 0$, so t = 1
Answer: (a) 10 m s^{-1} (b) t = 1

2 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 6, |b| = 7 and the angle between a and b is 90°.
Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 90° = 6 \times 7 \times 0 = 0$
$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 36 + 0 + 49 = 85$
$|\mathbf{a} + \mathbf{b}| = \sqrt{85}$
Answer: $\mathbf{a}\cdot\mathbf{b} = 0$, $|\mathbf{a} + \mathbf{b}| = \sqrt{85}$

3 Combined inequalities (All boards)

(a) Solve $3x - 3 \leq -2$.
(b) Solve $x^2 + 4x - 5 < 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $3x - 3 \leq -2$ so $x \leq \frac{1}{3}$
(b) $(x + 5)(x - 1) < 0$: critical values -5 and 1
(b) gives $-5 < x < 1$
(c) Overlap of the two sets.
Answer: (a) $x \leq \frac{1}{3}$ (b) $-5 < x < 1$ (c) $-5 < x \leq \frac{1}{3}$

4 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 12 kg box is pulled from rest along rough horizontal ground by a horizontal force of 50.64 N and moves 9.6 m in 8 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $9.6 = \frac{1}{2}a \times 8^2$, so a = 0.3
(a) $P - \mu 12g = 12a$: $50.64 - 117.6\mu = 3.6$, so $\mu = 0.4$
(b) $v = at = 0.3 \times 8 = 2.4$; after removal deceleration = $\mu g = 3.92$
$0 = 2.4^2 - 2 \times 3.92 \times s$, $s = 0.735$
Answer: (a) $\mu = 0.4$ (b) 0.735 m

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = -x^2 + x + 5$ is transformed by a reflection in the x-axis, followed by a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Find the equation of the image curve.

Reflect in x-axis: y becomes −y: $y = x^2 - x - 5$
Translation: replace x by $(x - 3)$ and change y by −1: $y = x^2 - 7x + 6$
Answer: $y = x^2 - 7x + 6$

6 Separable differential equations (All boards)

Solve the differential equation

$x\frac{dy}{dx} = 3y$,

given that $y = 2$ when $x = 1$ ($x > 0$, $y > 0$). Give y in terms of x.

Separate: $\int \frac{1}{y}\,dy = \int \frac{3}{x}\,dx$, so $\ln y = 3 \ln x + C$

$x = 1$, $y = 2$ gives $C = \ln 2$

$\ln y = \ln(2x^3)$, so $y = 2x^3$

Answer: $y = 2x^3$

7 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = -2u_n + 1$.

Given that $u_3 = -17$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = -2k + 1$, $u_3 = (-2)(-2k + 1) + 1$

$4k - 1 = -17$ so $k = -4$

Terms: $-4, 9, -17, 35$

Sum = 23

Answer: (a) $k = -4$ (b) 23

8 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(58, 4^2)$. Find $P(X > 61 \mid X > 59)$ to 3 significant figures.

Since $X > 61$ implies $X > 59$: $P = \frac{P(X > 61)}{P(X > 59)}$

$P(X > 61) = P(Z > 0.750) = 0.227$

$P(X > 59) = P(Z > 0.250) = 0.401$

$P = \frac{0.227}{0.401} = 0.565$

Answer: 0.565

9 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 5 \end{pmatrix}$ and $\begin{pmatrix} k \\ -1 \\ -4 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - k - 20 = 0$

$(k - 5)(k + 4) = 0$

$k = -4$ or $k = 5$. For $k = 5$ the first vector is $\begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$, with length $\sqrt{75} = 5\sqrt{3}$

Answer: $k = -4$ or 5; length $5\sqrt{3}$

10 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(30, p)$. H_0: $p = 0.2$ is tested against H_1: $p \neq 0.2$ at the 5% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. $P(X \leq 1) = 0.0105$, $P(X \leq 2) = 0.0442$

$P(X \geq 12) = 0.00949$, $P(X \geq 11) = 0.0256$

Critical region: $X \leq 1$ or $X \geq 12$

(b) Actual level = 0.0105 + 0.00949 = 0.0200

Answer: (a) $X \leq 1$ or $X \geq 12$ (b) 0.0200