Day 259 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $3^{3x-1} = 7^{x-2}$.
Give your answer to 3 significant figures.

Take logs: $(3x - 1)\ln 3 = (x - 2)\ln 7$
Collect x terms: $x(3\ln 3 - \ln 7) = -2\ln 7 + \ln 3$
$x = \dfrac{-2\ln 7 + \ln 3}{3\ln 3 - \ln 7}$
$x = -2.07$
Answer: $x = -2.07$

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 280 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $280 = 6\times20 + \frac{1}{2}\times a\times20^2$, so $a = 0.8$
(b) $v = u + at = 6 + 0.8\times20 = 22$
(c) For the midpoint, $s = 140$: $v^2 = u^2 + 2as = 6^2 + 2\times0.8\times140$
$v^2 = 260$
Answer: (a) 0.8 m s^{-2} (b) 22 m s^{-1} (c) 16.1 m s^{-1}

3 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 3 that lie strictly between 138 and 455.

First multiple 141, last multiple 453, common difference 3.

Number of terms $n = \dfrac{453 - 141}{3} + 1 = 105$

$S = \dfrac{105}{2}(141 + 453) = 31185$

Answer: 31185

4 Arc length and sector area (All boards)

A sector of a circle of radius 5 cm has perimeter 13 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

Perimeter $= 2r + r\theta$: $13 = 10 + 5\theta$

$\theta = \dfrac{3}{5} = 0.6$

Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}\times5^2\times0.6 = 7.5$

Answer: (a) 0.6 rad (b) 7.5 cm^2

5 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(3\mathbf{i} - 2\mathbf{j})$ m s^{-2}. When t = 0, P is at $(3\mathbf{i} - \mathbf{j})$ m moving with velocity $(-9\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find (a) when P moves parallel to $\mathbf{j}$, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-9\mathbf{i} + 2\mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j})$
(a) Parallel to $\mathbf{j}$: the $\mathbf{i}$ component is zero: $-9 + 3t = 0$, so $t = 3$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$= (3\mathbf{i} - \mathbf{j}) + 3(-9\mathbf{i} + 2\mathbf{j}) + 4.5(3\mathbf{i} - 2\mathbf{j})$
$= -10.5\mathbf{i} - 4\mathbf{j}$
Answer: (a) $t = 3$ (b) $\mathbf{r} = (-10.5\mathbf{i} - 4\mathbf{j})$ m

Day 259 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 18 and the coefficient of x^2 is 135. Find n and k.

$nk = 18$ and $\frac{n(n-1)}{2}k^2 = 135$

$(n-1)k = \frac{2 \times 135}{18} = 15$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 18 - 15 = 3$

$n = \frac{18}{3} = 6$

Answer: n = 6, k = 3

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is $N(70, 5^2)$. An egg is classed as 'special' if its mass is less than 65. 11 eggs are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

$P(\text{special}) = P(X < 65) = P(Z < -1.00) = 0.159$

$Y \sim B(11, 0.159)$

$P(Y = 3) = C(11, 3)(0.159)^3(0.841)^8 = 0.165$

Answer: 0.165

8 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 7 \\ -6 \\ -1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(3, 2, −1) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} -2t + 4 \\ 2t - 8 \\ 0 \end{pmatrix}$

$\mathrm{PF} \cdot \mathbf{d} = 0$ gives $8t - 24 = 0$, so $t = 3$

F = (1, 0, −1); $\mathrm{PF} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{8} = 2\sqrt{2}$

Answer: F(1, 0, −1), distance $2\sqrt{2}$

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{x - 1}{(1 + 2x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 2x)} + \frac{B}{(1 + x)}$: $A = -3$, $B = 2$

(b) $-3(1 + 2x)^{-1} = -3(1 - 2x + 4x^2 \ldots)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|x| < 1$

Answer: (a) $\frac{-3}{(1 + 2x)} + \frac{2}{(1 + x)}$ (b) $-1 + 4x - 10x^2$ (c) $|x| < \frac{1}{2}$

10 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 58) = 0.0228$ and $P(X > 73) = 0.3085$.
Find μ and σ.

z-values: 58: $z = -2.00$; 73: $z = 0.500$
$58 = \mu - 2.00\,\sigma$ and $73 = \mu + 0.500\,\sigma$
Subtract: $15 = 2.50\,\sigma$, so $\sigma = 6.00$
$\mu = 58 + 2.00 \times 6.00 = 70.0$
Answer: $\mu = 70$, $\sigma = 6$