Day 258 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 90 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $90 = 6\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.6$
(b) $v = u + at = 6 + 0.6\times10 = 12$
(c) For the midpoint, $s = 45$: $v^2 = u^2 + 2as = 6^2 + 2\times0.6\times45$
$v^2 = 90$

Answer: (a) 0.6 m s^{-2} (b) 12 m s^{-1} (c) 9.49 m s^{-1}

2 Discriminant: range of k (All boards)

The equation $x^2 + (k + 3)x + (k + 6) = 0$ has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k + 3)^2 - 4(k + 6)$
$= k^2 + 2k - 15$
Roots of the quadratic in k: $k = -5$ and $k = 3$
Required: $b^2 - 4ac < 0$
Answer: $-5 < k < 3$

3 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 3 seconds before it hits the sea it falls 249.9 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 3)$ s the stone falls $4.9(t - 3)^2$.
$4.9t^2 - 4.9(t - 3)^2 = 249.9$
$4.9(6t - 9) = 249.9$, so $t = 10$
Height $= 4.9\times10^2 = 490$
Answer: 490 m

4 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 3$, $y = -2t(t^2 - 9)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$
$t = 0$: $x = -3$
$t = \pm3$: $x = 24$
Answer: $(-3, 0)$ and $(24, 0)$

5 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 6 men and 8 women. A team of 3 is chosen at random.
Find the probability that the team has exactly 2 of the women.

Total teams = C(14, 3) = 364
Favourable = C(8, 2) × C(6, 1) = 28 × 6 = 168
$P = \frac{168}{364}$
$= \frac{6}{13}$
Answer: $\frac{6}{13}$

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6 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^2 x\,e^{-x}\,dx$.

$u = x$, $\frac{dv}{dx} = e^{-x}$: $\int xe^{-x}\,dx = [-\frac{xe^{-x}}{1}] + \int \frac{e^{-x}}{1}\,dx$

$= [-\frac{xe^{-x}}{1} - \frac{e^{-x}}{1}]$

$= 1 - 3e^{-2}$

Answer: $1 - 3e^{-2}$

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{15} in the expansion of

$(2x^3 + \frac{2}{x})^9$.

General term: ${}^9C_r\,(2x^3)^{9-r}\,(2x^{-1})^r$

Power of x: $3(9 - r) - 1r = 15$

$27 - 4r = 15$ so $r = 3$

Coefficient = ${}^9C_3 \times 2^6 \times 2^3 = 43008$

Answer: 43008

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - 2xy + 4y^2 = 3$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - 2y}{-2x + 8y}$

Horizontal tangent: $2x - 2y = 0$, so $y = x$

Substitute: $3x^2 = 3$, so $x^2 = 1$ and $x = \pm 1$

Points: (1, 1) and (−1, −1)

Answer: (1, 1) and (−1, −1)

9 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 20 cm^3s^{-1}. The surface radius is always $\frac{h}{4}$.

Find the rate at which the depth is rising when h = 5 cm.

$V = \frac{1}{3}\pi(h/4)^2h = \frac{\pi h^3}{48}$, so $\frac{dV}{dh} = \frac{\pi h^2}{16}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{20 \times 16}{\pi h^2}$

$h = 5$: $\frac{dh}{dt} = \frac{64}{5\pi}$ cm s^{-1}

Answer: $\frac{64}{5\pi}$ cm/s

10 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 63 and standard deviation 5.

Find (a) P(X > 74.5) (b) P(74.5 < X < 82), each to 3 s.f.

(a) $z = \frac{74.5 - 63}{5} = 2.30$, P(Z > 2.30) = 0.0107

(b) $z_1 = 2.30$, $z_2 = \frac{82 - 63}{5} = 3.80$

$P(74.5 < X < 82) = \Phi(3.80) - \Phi(2.30) = 0.0107$

Answer: (a) 0.0107 (b) 0.0107