**Day 257** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Geometric distribution (CIE)

On each independent trial, the probability that a biased die shows a six is $\frac{1}{4}$.

X is the number of trials up to and including the first success.
(a) Find P(X = 5) and E(X).
(b) Find the least m with P(X ≤ m) > 0.99.

(a) $P(X = 5) = (\frac{3}{4})^4 \times \frac{1}{4} = \frac{81}{1024}$; $E(X) = \frac{1}{p} = 4$

(b) $P(X \leq m) = 1 - (\frac{3}{4})^m > 0.99$

$(\frac{3}{4})^m < 0.01$, so $m > 16.0$

Least integer m = 17

**Answer: (a) $\frac{81}{1024}$, E(X) = 4 (b) m = 17**

## 2 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 4 m $s^{-1}$. 6 s later car B leaves P from rest, in the same direction, with acceleration 1 m $s^{-2}$. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 6) seconds.
B: ½×1×$s^2$ = 0.5 $s^2$ A: 4(s + 6)
0.5 $s^2$ − 4 s − 24 = 0 so s = 12 (s > 0)
(b) PQ = 0.5×$12^2$ = 72
(c) v = 0 + at = 1×12 = 12
**Answer: (a) 12 s (b) 72 m (c) 12 m $s^{-1}$**

## 3 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x + 8\cot x - 1 = 0$
for 0° < x < 360°. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x + 8\cot x + 3 = 0$

$\cot x = -\frac{3}{2}$ or $\cot x = -\frac{1}{2}$

$\cot x = -\frac{3}{2}$ means $\tan x = -\frac{2}{3}$: x = 146.3°, 326.3°

$\cot x = -\frac{1}{2}$ means $\tan x = -2$: x = 116.6°, 296.6°

**Answer: x = 116.6°, 146.3°, 296.6°, 326.3°**

## 4 Arithmetic series: finding n (All boards)

An arithmetic series has first term −7 and common difference 3.
The sum of the first n terms is 333. Find n.

$\frac{n}{2}(2 \times (-7) + (n - 1) \times 3) = 333$

$3n^2 - 17n = 666$

$3n^2 - 17n - 666 = 0$

$(n - 18)(3n + 37) = 0$, so n = 18 or $n = -\frac{37}{3}$

n must be a positive integer, so n = 18
**Answer: 18**

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## 5 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\ln 5} (2e^{2x} + 4e^x)\,dx$.

Integrate: $[e^{2x} + 4e^x]$
Substitute the limits and subtract: 40
**Answer: 40**

## 6 Conservation of momentum (CIE)

Particles A (2 kg) and B (8 kg) move in the same direction on a smooth horizontal line at 3 m s$^{-1}$ and 5 m s$^{-1}$. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
$2\times3 + 8\times5 = 10v$, so $v = 4.6$
KE before = $\frac{1}{2}\times2\times3^2 + \frac{1}{2}\times8\times5^2 = 109$
KE after = $\frac{1}{2}\times10\times4.6^2 = 105.8$
Loss = $109 - 105.8$
**Answer: (a) 4.6 m s$^{-1}$ (b) 3.2 J**

## 7 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(20, p)$. $H_0$: $p = 0.2$ is tested against $H_1$: $p \neq 0.2$ at the 5% level.
(a) Find the critical region.
(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail $\leq 0.025$. $P(X \leq 0) = 0.0115$, $P(X \leq 1) = 0.0692$
$P(X \geq 9) = 0.00998$, $P(X \geq 8) = 0.0321$
Critical region: $X \leq 0$ or $X \geq 9$
(b) Actual level = 0.0115 + 0.00998 = 0.0215
**Answer: (a) $X \leq 0$ or $X \geq 9$ (b) 0.0215**

## 8 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 6x + 8y - 75 = 0$ passes through P(9, 4).
(a) Find the equation of the tangent at P, in the form $ax + by = c$.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre $(3, -4)$. Gradient of radius CP = $\frac{4}{3}$

Gradient of tangent = $-\frac{3}{4}$

$y - 4 = -\frac{3}{4}(x - 9)$: $3x + 4y = 43$

$y = 0$: $x = \frac{43}{3}$

**Answer: (a) $3x + 4y = 43$ (b) $(\frac{43}{3}, 0)$**

## 9 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{22 + 15\sin\theta + 8\cos\theta}$
(a) Write $15\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, with $\alpha$ acute.
(b) Find the greatest and least values of $g(\theta)$ and the smallest positive $\theta$ at which the greatest value occurs (1 d.p.).

$R = \sqrt{15^2 + 8^2} = 17$, $\alpha = \tan^{-1}\frac{8}{15} = 28.1°$

Denominator = $22 + 17\sin(\theta + 28.1°)$, ranging from 5 to 39

Greatest g = $\frac{1}{5}$ (denominator least), least g = $\frac{1}{39}$

Least denominator when $\sin(\theta + 28.1°) = -1$: $\theta + 28.1° = 270°$
$\theta = 241.9°$

**Answer: R = 17, $\alpha$ = 28.1°; greatest $\frac{1}{5}$, least $\frac{1}{39}$; $\theta$ = 241.9°**

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## 10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-21x + 8}{(1 - 2x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to $x^2$.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 - 3x)}$: A = 5, B = 3

(b) $5(1 - 2x)^{-1} = 5(1 + 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need |2x| < 1 and |3x| < 1

**Answer: (a)** $\frac{5}{(1 - 2x)} + \frac{3}{(1 - 3x)}$ **(b)** $8 + 19x + 47x^2$ **(c)** $|x| < \frac{1}{3}$