Day 256 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Partial fractions: distinct linear factors (All boards)

Express $\frac{-5x + 8}{(x - 2)(x - 4)}$ in partial fractions.

Let $\frac{-5x + 8}{(x - 2)(x - 4)} \equiv \frac{A}{(x - 2)} + \frac{B}{(x - 4)}$

$-5x + 8 \equiv A(x - 4) + B(x - 2)$

Substitute $x = 2$: $A = 1$

Substitute $x = 4$: $B = -6$

Answer: $\mathbf{\frac{1}{(x - 2)} + \frac{-6}{(x - 4)}}$

2 Index equations solved with logarithms (All boards)

Solve $3^{3x + 3} = 2^{2x + 1}$.

Give your answer to 3 significant figures.

Take logs: $(3x + 3) \ln 3 = (2x + 1) \ln 2$

Collect x terms: $x(3 \ln 3 - 2 \ln 2) = \ln 2 - 3 \ln 3$

$x = \frac{\ln 2 - 3 \ln 3}{3 \ln 3 - 2 \ln 2}$

$x = -1.36$

Answer: $x = -1.36$

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 850 items.

The 3th item chosen is number 65 and the 8th item chosen is number 190.

Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(190 - 65) \div (8 - 3) = 25$

First item = $65 - 2 \times 25 = 15$

Sample size = $850 \div 25 = 34$

Answer: Interval 25, first 15, size 34

4 Fixed point iteration (All boards)

The equation $e^x = 3x + 1$ has a root α. Use the iteration $x_{n+1} = \ln(3x + 1)$ with $x_0 = 2$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(3 \times 2 + 1) = 1.9459$

$x_2 = 1.9225$, $x_3 = 1.9121$

Continuing, the values settle at 1.9038, so $\alpha = 1.90$

Answer: $x_1 = 1.9459$, $x_2 = 1.9225$, $x_3 = 1.9121$; $\alpha = 1.90$

5 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 10 kg box is pulled from rest along rough horizontal ground by a horizontal force of 35.4 N and moves 19.2 m in 8 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $19.2 = \frac{1}{2}a \times 8^2$, so $a = 0.6$

(a) $P - \mu 10\,g = 10\,a$: $35.4 - 98\,\mu = 6$, so $\mu = 0.3$

(b) $v = at = 0.6 \times 8 = 4.8$; after removal deceleration $= \mu g = 2.94$

$0 = 4.8^2 - 2 \times 2.94 \times s$, $s = 3.92$

Answer: (a) $\mu = 0.3$ (b) 3.92 m

6 Independent and mutually exclusive events (All boards)

P(A) = 0.5 and P(A ∪ B) = 0.9.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) P(A ∪ B) = P(A) + P(B) − P(A)P(B), so 0.9 = 0.5 + P(B)(1 − 0.5)

$P(B) = \frac{0.9 - 0.5}{0.5} = 0.8$

(b) P(A ∩ B) = 0, so P(B) = 0.9 − 0.5 = 0.4
(c) P(A′ ∩ B) = P(A′)P(B) = 0.5 × 0.8 = 0.4
Answer: (a) 0.8 (b) 0.4 (c) 0.4

7 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 5 m, mass 6 kg) has its centre of mass at G, AG = 2 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 3 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: 2T = (6 + m)g, so T = ½(6 + m)g
Moments about A: T×5 = 6 g×2 + m g×3
½(6 + m)×5 = 12 + 3 m → 15 + 2.5 m = 12 + 3 m
m (3 − 2.5) = 15 − 12, so m = 6
(b) T = ½(6 + 6)×9.8
Answer: (a) m = 6 (b) 58.8 N

8 Partial fractions: repeated factor (All boards)

Express $\frac{x^2 - 3x + 4}{(x-3)(x-2)^2}$ in the form $\frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$, and find A, B and C.

$x^2 - 3x + 4 \equiv A(x-2)^2 + B(x-3)(x-2) + C(x-3)$
x = 3: $4 = A(1)^2$ so A = 4
x = 2: 2 = C(−1) so C = −2
Coefficient of x^2: 1 = A + B so B = −3
Answer: A = 4, B = −3, C = −2

9 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−2, 0) and passes through B(−4, 2).
Find the equation of the circle.

Touches the x-axis at A, so the centre is (−2, r) and the radius is r.
B on the circle: $(-2)^2 + (2 - r)^2 = r^2$
4 + 4 − 4r = 0 so r = 2
Answer: $(x + 2)^2 + (y - 2)^2 = 4$

10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 6 \\ -6 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(3, 0, 0) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} 3t + 3 \\ -3t - 6 \\ 3t \end{pmatrix}$

PF·d = 0 gives 27t + 27 = 0, so t = −1

F = (3, −3, −3); $\mathrm{PF} = \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{18} = 3\sqrt{2}$

Answer: F(3, −3, −3), distance $3\sqrt{2}$