Day 255 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 44.8 m ahead. After a reaction time of 0.8 s she brakes with deceleration 4 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.8 v, braking distance = $v^2 \div (2\times4)$

$0.8\,v + v^2 \div 8 = 44.8$, so $v^2 + 6.4\,v - 358.4 = 0$

$(v - 16)(v + 22.4) = 0$, so $v = 16$

(b) Braking time = $16 \div 4 = 4$ s; total = 0.8 + 4

Answer: (a) 16 m s^{-1} (b) 4.8 s

2 Completing the square (All boards)

$f(x) = 2x^2 + 20x + 45$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = 2(x + 5)^2 - 5$

(b) minimum value -5 when $x = -5$

(c) $2(x + 5)^2 = 5$, so $(x + 5)^2 = \frac{5}{2}$

$x + 5 = \pm\sqrt{\frac{5}{2}}$

Answer: (a) $2(x + 5)^2 - 5$ (b) minimum -5 at $x = -5$ (c) $x = -5 \pm \frac{\sqrt{10}}{2}$

3 Arc length and sector area (All boards)

A sector of a circle of radius 8 cm has angle 240°.

Find, in terms of π, the arc length, the area and the perimeter of the sector.

$\theta = 240° = \frac{4\pi}{3}$ radians

Arc length $= r\theta = 8 \times \frac{4\pi}{3} = \frac{32\pi}{3}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{4\pi}{3} = \frac{128\pi}{3}$ cm^2

Perimeter = 2r + arc $= \frac{32\pi + 48}{3}$ cm

Answer: arc $\frac{32\pi}{3}$ cm, area $\frac{128\pi}{3}$ cm^2, perimeter $\frac{32\pi + 48}{3}$ cm

4 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.

(a) Find the equation of the line through C perpendicular to AB.

(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)

Through C: $x = 2$

Line AB: $y - 3 = 0(x - -4)$. Solve to get D = (2, 3)

Area = ½ × AB × CD $= \frac{33}{2}$

Answer: (a) $x = 2$ (b) D(2, 3), area $\frac{33}{2}$

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5 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{\sin 4x \tan 3x}{1 - \cos 2x}$ approaches as $x \to 0$.

$\sin 4x \approx 4x$, $\tan 3x \approx 3x$, $1 - \cos 2x \approx \frac{1}{2}(2x)^2$

Expression $\approx \frac{4x \times 3x}{\frac{4}{2}x^2} = \frac{12x^2}{\frac{4}{2}x^2} = 6$

Answer: 6

6 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(35, p)$. H_0: $p = 0.5$ is tested against H_1: $p < 0.5$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.3$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 12$
(b) Type I error = P(reject H_0 | $p = 0.5$) = 0.0448
(c) Type II error = P(not in critical region | $p = 0.3$)
$P(X \geq 13)$ when $p = 0.3$: 0.227
Answer: (a) $X \leq 12$ (b) 0.0448 (c) 0.227

7 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 4\sin t$, $y = 5\cos 2t$.
Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -\frac{10\sin 2t}{4\cos t}$

$t = \frac{\pi}{4}$: gradient $m = -\frac{5\sqrt{2}}{2}$, point $(2\sqrt{2}, 0)$

$c = y - mx = 10$

Answer: $\frac{dy}{dx} = -\frac{10\sin 2t}{4\cos t}$; $y = -\frac{5\sqrt{2}}{2}x + 10$

8 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 900 kg car tows a 200 kg trailer by a light tow-bar on a horizontal road. Driving force 830 N; resistances 400 N (car), 100 N (trailer). Find (a) the acceleration and the tow-bar tension. At 10 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $830 - 400 - 100 = 1100\,a$, so $a = 0.3$
Trailer: $T - 100 = 200\,a$, so $T = 100 + 200 \times 0.3 = 160$
(b) After the break the trailer decelerates at $100 \div 200 = 0.5$ m s^{-2}
$0 = 10^2 - 2 \times 0.5 \times s$, so $s = 100$
Answer: (a) 0.3 m s^{-2}, 160 N (b) 100 m

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.15, independently. A tray of 8 eggs is rejected if it has at least 2 defective. From 9 such trays, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(8, 0.15)$. P(rejected) = $P(Y \geq 2) = 1 - 0.657 = 0.343$
$W \sim B(9, 0.343)$ is the number rejected
$P(W = 2) = C(9, 2)(0.343)^2(1 - 0.343)^7 = 0.224$
Answer: 0.224

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that 10m + 15n = 107.

Assume 10m + 15n = 107 for integers m, n.
Then 5(2m + 3n) = 107, so 5 divides 107.
But 107 is not a multiple of 5. Contradiction.
Answer: no such integers exist