Day 254 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}-4\\1\end{pmatrix}$ and $\begin{pmatrix}-4\\6\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 4.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{1}{5}\text{AB} = \begin{pmatrix}-4\\1\end{pmatrix} + \frac{1}{5}\begin{pmatrix}0\\5\end{pmatrix}$

$= \begin{pmatrix}-4\\2\end{pmatrix}$

$\text{OP} = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

Answer: $\begin{pmatrix}\mathbf{-4}\\\mathbf{2}\end{pmatrix}$**, OP =** $\mathbf{2\sqrt{5}}$

2 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, μ = 0.4. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, weight W = 19.6 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 11.9$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 72.8$

Answer: least 11.9 N, greatest 72.8 N

3 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of NUMBERS have N and U not next to each other?

All arrangements = 7! = 5040

N and U together: 2 × 6! = 1440

Not together = 5040 − 1440 = 3600

Answer: 3600

4 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-7\mathbf{i} - \mathbf{j})$ N, $F_2 = (5\mathbf{i} - 12\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 4 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

i: −7 + 5 + a = 0, so a = 2

j: −1 + (−12) + b = 0, so b = 13

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{5^2 + -12^2} = 13$ N

$a = 13 \div 4$

Answer: (a) a = 2, b = 13 (b) 3.25 m s^{-2}

5 Partial fractions and integration (All boards)

(a) Express $\frac{3x + 16}{(x + 4)^2}$ in the form $\frac{A}{x + 4} + \frac{B}{(x + 4)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{3x + 16}{(x + 4)^2}\,dx$.

$3x + 16 \equiv A(x + 4) + B$, so A = 3 and B = 4

$I = [3\ln(x + 4) - \frac{4}{x + 4}]_0^4$

$= 3\ln\frac{8}{4} + 4(\frac{1}{4} - \frac{1}{8})$

$= \frac{6\ln 2 + 1}{2}$

Answer: (a) A = 3, B = 4; (b) $\mathbf{\frac{6\ln 2 + 1}{2}}$

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6 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 + 6x - 4$ is transformed by a translation of $\begin{pmatrix}3\\2\end{pmatrix}$, followed by a reflection in the x-axis.

Find the equation of the image curve.

Translation: replace x by $(x - 3)$ and change y by 2: $y = x^2 - 11$

Reflect in x-axis: y becomes $-y$: $y = -x^2 + 11$

Answer: $y = -x^2 + 11$

7 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 14\cot x - 12 = 0$
for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 14\cot x - 8 = 0$

$\cot x = -\frac{1}{2}$ or $\cot x = 4$

$\cot x = -\frac{1}{2}$ means $\tan x = -2$: $x = 116.6°$

$\cot x = 4$ means $\tan x = \frac{1}{4}$: $x = 14.0°$

Answer: $x = 14.0°, 116.6°$

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x + 4y = -14$ meets the circle $(x + 4)^2 + (y + 4)^2 = 16$ at the points A and B. Find the exact length of AB.

Centre $(-4, -4)$, radius 4. Distance from centre to line:

$d = \frac{|-14|}{5} = \frac{14}{5}$

$\text{Half-chord}^2 = 16 - (\frac{14}{5})^2 = \frac{204}{25}$

$AB = 2\sqrt{\frac{204}{25}}$

Answer: $\frac{20\sqrt{51}}{25}$

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.15, independently. A tray of 8 eggs is rejected if it has at least 3 defective. From 9 such trays, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(8, 0.15)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.895 = 0.105$

$W \sim B(9, 0.105)$ is the number rejected

$P(W = 3) = C(9, 3)(0.105)^3(1 - 0.105)^6 = 0.0502$

Answer: 0.0502

10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 2) = 5 - 4\sqrt{5}$, writing x in the form $p + q\sqrt{5}$.

$x = \frac{5 - 4\sqrt{5}}{\sqrt{5} - 2}$

Multiply top and bottom by $\sqrt{5} + 2$.

Numerator: $(5 - 4\sqrt{5})(\sqrt{5} + 2) = -10 - 3\sqrt{5}$

Denominator: $5 - 4 = 1$

Answer: $x = -10 - 3\sqrt{5}$