Day 253 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 660 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(100 + 65) × V
660 = 82.5 V, so V = 8
(b) Deceleration = V ÷ 20 = 8 ÷ 20
Answer: (a) V = 8 (b) 0.4 m s^{-2}

2 Fixed point iteration (All boards)

The equation $e^x = 4x + 2$ has a root α. Use the iteration $x_{n+1} = \ln(4x + 2)$ with $x_0 = 3$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(4 \times 3 + 2) = 2.6391$
$x_2 = 2.5302$, $x_3 = 2.4949$
Continuing, the values settle at 2.4773, so α = 2.48
Answer: x_1 = 2.6391, x_2 = 2.5302, x_3 = 2.4949; α = 2.48

3 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 800 kg car tows a 250 kg trailer by a light tow-bar on a horizontal road. Driving force 965 N; resistances 400 N (car), 250 N (trailer). Find (a) the acceleration and the tow-bar tension. At 8 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: 965 − 400 − 250 = 1050 a, so a = 0.3
Trailer: T − 250 = 250 a, so T = 250 + 250×0.3 = 325
(b) After the break the trailer decelerates at 250 ÷ 250 = 1 m s^{-2}
$0 = 8^2 - 2 \times 1 \times s$, so s = 32
Answer: (a) 0.3 m s^{-2}, 325 N (b) 32 m

4 Algebraic fractions (All boards)

Simplify fully $\dfrac{4x^2 + 4x - 15}{4x^2 - 2x - 6}$.

Factorise the numerator: (2x − 3)(2x + 5)
Factorise the denominator: (2x − 3)(2x + 2)
Cancel the common factor (2x − 3).
Answer: $\dfrac{2x + 5}{2x + 2}$

5 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^4 \ln(x + 2)\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 0, 1, 2, 3, 4: 0.6931, 1.0986, 1.3863, 1.6094, 1.7918
$T = \frac{1}{2}[0.6931 + 1.7918 + 2(1.0986 + 1.3863 + 1.6094)]$
T = 5.34
The curve is concave (y'' < 0) on this interval, so the chords lie below the curve: underestimate
Answer: 5.34, an underestimate

6 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 2n^2 + 5n$.

(a) Find the first term and the common difference.
(b) Find u_{19}.

$u_1 = S_1 = 7$
$S_2 = 18$, so $u_2 = 11$
$d = u_2 - u_1 = 4$
$u_{19} = 7 + 18 \times 4$
Answer: (a) a = 7, d = 4 (b) 79

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7 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$3\sin 3\theta + 4\cos\theta = 7$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$3 \times 3\theta + 4(1 - \frac{1}{2}\theta^2) = 7$

Multiply by 2 and rearrange: $4\theta^2 - 18\theta + 6 = 0$

Divide through by 2: $2\theta^2 - 9\theta + 3 = 0$

$\theta = \frac{9 \pm \sqrt{57}}{4}$, so $\theta = 0.363$

Answer: a = 2, b = −9, c = 3; $\theta \approx 0.363$

8 Hypothesis test for a Poisson mean (CIE)

Complaints occur as a Poisson process at a mean rate of 3 per day.
After a change, the number recorded in one day is 6.
Test at the 1% level whether the mean rate has increased.

$H_0: \lambda = 3$, $H_1: \lambda > 3$; $X \sim Po(3)$

$P(X \geq 6) = 1 - P(X \leq 5) = 0.0839$

$0.0839 > 0.01$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.0839; do not reject H_0

9 Hypothesis test for a normal mean (All boards)

The mean mass of a sack (kg) is normally distributed with standard deviation 4; it was previously 25.
A random sample of 36 has mean 25.6.
Test, at the 5% level, whether the mean has increased.

$H_0: \mu = 25$, $H_1: \mu > 25$

$\bar{X} \sim N(25, \frac{4^2}{36})$, $z = \frac{25.6 - 25}{\frac{4}{6}} = 0.900$

p-value = 0.184 > 0.05

Do not reject H_0: no evidence that the mean has increased.

Answer: z = 0.900, p = 0.184; do not reject H_0

10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.
(b) Use $x = 0.025$ to estimate $\sqrt{105}$ to 3 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = 2x$

(a) $1 + x - \frac{1}{2}x^2$

$x = \frac{1}{40}$: $105 = 10^2(1 + \frac{1}{20})$ so $\sqrt{105} = 10 \times (1 + \frac{1}{20})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{40} - \frac{1}{3200}) = 10.246875$

Answer: (a) $1 + x - \frac{1}{2}x^2$ (b) 10.247