Day 252 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3 + 5\cos\theta$, $y = -4 + 5\sin\theta$.
Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x-3}{5}$, $\sin\theta = \frac{y+4}{5}$

Use $\cos^2\theta + \sin^2\theta = 1$: $(x-3)^2 + (y+4)^2 = 25$

Answer: $(x-3)^2 + (y+4)^2 = 25$, a circle with centre $(3, -4)$, radius 5

2 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{5}{13}$ and $\cos B = -\frac{24}{25}$.
Find the exact values of sin(A+B) and tan(A−B).

$\cos A = \frac{12}{13}$ (A acute), $\sin B = \frac{7}{25}$ (B obtuse), so $\tan A = \frac{5}{12}$, $\tan B = -\frac{7}{24}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B = \frac{5}{13} \times (-\frac{24}{25}) + \frac{12}{13} \times \frac{7}{25} = -\frac{36}{325}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} = \frac{\frac{5}{12} - (-\frac{7}{24})}{1 + \frac{5}{12} \times (-\frac{7}{24})} = \frac{204}{253}$

Answer: $\sin(A+B) = -\frac{36}{325}$, $\tan(A-B) = \frac{204}{253}$

3 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 values of x, $\Sigma(x-20) = 100$ and $\Sigma(x-20)^2 = 500$.
Find the mean and standard deviation of x.

Let $y = x - 20$: $\bar{y} = \frac{100}{25} = 4$

Mean of $x = 4 + 20 = 24$

Variance $= \frac{500}{25} - 4^2 = 4$

Subtracting a constant does not change spread, so sd = 2

Answer: mean 24, sd 2

4 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that
$\frac{\sin 3\theta \tan 4\theta}{1 - \cos 4\theta}$
approaches as $\theta \to 0$.

$\sin 3\theta \approx 3\theta$, $\tan 4\theta \approx 4\theta$, $1 - \cos 4\theta \approx \frac{1}{2}(4\theta)^2$

Expression $\approx \frac{3\theta \times 4\theta}{\frac{16}{2}\theta^2} = \frac{12\theta^2}{\frac{16}{2}\theta^2} = \frac{3}{2}$

Answer: $\frac{3}{2}$

5 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $5x + 12y = k$ is a tangent to the circle
$(x-1)^2 + (y-2)^2 = 4$.
Find the two possible values of k.

Centre (1, 2), radius 2

Distance from centre to the line = radius:

$\frac{|29 - k|}{13} = 2$

$|29 - k| = 26$

Answer: k = 3 or k = 55

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6 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(24, 0.25).
Find the largest value of c such that $P(X \leq c) < 0.01$.

$P(X \leq 1) = 0.00903$
$P(X \leq 2) = 0.0398$
$0.00903 < 0.01$ but $0.0398 > 0.01$
Answer: c = 1

7 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (12, −6). Its centre lies in the fourth quadrant. Find the equations of the two possible circles.

Centre (r, −r), radius $r > 0$.
Through (12, −6): $(12 - r)^2 + (-6 + r)^2 = r^2$
$r^2 - 36r + 180 = 0$
$(r - 6)(r - 30) = 0$, so r = 6 or 30
Answer: $(x - 6)^2 + (y + 6)^2 = 36$ or $(x - 30)^2 + (y + 30)^2 = 900$

8 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 15 kg box is pulled from rest along rough horizontal ground by a horizontal force of 44.25 N and moves 25 m in 10 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $25 = \frac{1}{2}a \times 10^2$, so $a = 0.5$
(a) $P - \mu 15\,g = 15\,a$: $44.25 - 147\,\mu = 7.5$, so $\mu = 0.25$
(b) $v = at = 0.5 \times 10 = 5$; after removal deceleration $= \mu g = 2.45$
$0 = 5^2 - 2 \times 2.45 \times s$, $s = 5.10$
Answer: (a) μ = 0.25 (b) 5.10 m

9 Equations with sec x and tan x (All boards)

Solve $4\sec^2\theta + 6\tan\theta - 8 = 0$
for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $4\tan^2\theta + 6\tan\theta - 4 = 0$
$\tan\theta = -2$ or $\tan\theta = \frac{1}{2}$
$\tan\theta = -2$: $\theta = 116.6°, 296.6°$
$\tan\theta = \frac{1}{2}$: $\theta = 26.6°, 206.6°$
Answer: θ = 26.6°, 116.6°, 206.6°, 296.6°

10 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (12 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 2 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{3}{4}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 4 = 12\,g \times 2 + 2\,g \times 4$
$T\sin\theta = (6 + 2)g = 78.4$, so (a) $T = 131$
Horizontal component of hinge force: $X = T\cos\theta = 105$
Vertical: $Y + T\sin\theta = (12 + 2)g$, so $Y = 12\,g \div 2 = 58.8$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(105^2 + 58.8^2)}$
Answer: (a) 131 N (b) 120 N