Day 251 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 3rd term of an arithmetic sequence is 21 and the 7th term is 25.
Find the first term and common difference, and the sum of the first 20 terms.

$u_7 - u_3 = 4\,d = 25 - 21$ so $d = 1$

$a + 2\,d = 21$ so $a = 19$

$S_{20} = \frac{20}{2}(2 \times 19 + 19 \times 1)$

Answer: a = 19, d = 1, $S_{20} = 570$

2 Exponential equations reducing to quadratics (All boards)

Solve $3^{2x} - 233^{x} + 126 = 0$.
Give exact roots, or 3 s.f. where the answer is not exact.

Let $u = 3^x$: $u^2 - 23\,u + 126 = 0$, so $(u - 9)(u - 14) = 0$

$3^x = 9$ gives $x = 2$; $3^x = 14$ gives $x = \frac{\ln 14}{\ln 3} = 2.40$

Answer: x = 2 or x = 2.40

3 Algebraic fractions (All boards)

Simplify $\frac{x^2 - 4}{(3x - 4)} \div \frac{(x + 2)}{(3x + 2)}$.

$\frac{x^2 - 4}{(3x - 4)} \times \frac{(3x + 2)}{(x + 2)}$

$x^2 - 4 = (x - 2)(x + 2)$

Cancel $(x + 2)$.

Answer: $\frac{(x - 2)(3x + 2)}{(3x - 4)}$

4 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, $\mu = 0.4$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, $R = 3\,g\cos\alpha = \frac{441}{17}$ N

Limiting friction $= \mu R = \frac{882}{85}$ N

P smallest: friction acts up the slope: $P + \mu R = 3\,g\sin\alpha$, $P = \frac{1176}{85} - \frac{882}{85}$

P greatest: friction acts down the slope: $P = \frac{1176}{85} + \frac{882}{85}$

Answer: 3.46 N ≤ P ≤ 24.2 N

5 Discriminant: range of k (All boards)

The equation $x^2 + (k - 2)x + (k + 1) = 0$ has real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k - 2)^2 - 4(k + 1)$

$= k^2 - 8k$

Roots of the quadratic in k: $k = 0$ and $k = 8$

Required: $b^2 - 4ac \geq 0$

Answer: k ≤ 0 or k ≥ 8

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6 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = \frac{1}{t}$, $y = t^2$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where t = 1, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -2t^3$

t = 1: point (1, 1), gradient -2

Normal gradient = $-1 \div (-2) = \frac{1}{2}$, so normal: $x - 2y + 1 = 0$

Answer: $\frac{dy}{dx} = -2t^3$; $x - 2y + 1 = 0$

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = 0.65\, T_1$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 3\,g = 29.4$

$T_1(0.8 + 0.25) = 29.4$

$T_1 = 28.0$, $T_2 = 18.2$

Answer: $T_1 = 28$ N, $T_2 = 18.2$ N

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{4}{625}x^3$ for $0 \le x \le 5$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \le x \le 5$.

(b) Find $P(X > \frac{5}{2})$. (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^4}{625}$

(b) $P(X > \frac{5}{2}) = 1 - F(\frac{5}{2}) = \frac{15}{16}$

(c) $F(m) = \frac{1}{2}$: $m^4 = \frac{5^4}{2}$, so $m = 5 \times (0.5)^{\frac{1}{4}}$

m = 4.20

Answer: (b) $\frac{15}{16}$ **(c) 4.20**

9 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events A and B.

(a) Find P(B′).

(b) Find P(A′ | B′).

(a) $P(B') = 0.6 \times 0.1 + 0.4 \times 0.4 = 0.22$

(b) $P(A' \mid B') = \frac{0.4 \times 0.4}{0.22}$

Answer: (a) 0.22 (b) $\frac{8}{11}$

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10 Partial fractions and integration (All boards)

(a) Express $\frac{x}{(x+3)(x+2)}$ in partial fractions.

(b) Hence find $\int_0^5 \frac{x}{(x+3)(x+2)}\,dx$, giving the answer in the form ln k.

$x \equiv A(x+2) + B(x+3)$: $x = -2$ gives $A = 3$; $x = -3$ gives $B = -2$

$I = [3\ln(x+3) - 2\ln(x+2)]_0^5$

$= 3\ln\frac{8}{3} - 2\ln\frac{7}{2}$

$= \ln\frac{2048}{1323}$

Answer: (a) $\frac{3}{x+3} - \frac{2}{x+2}$; (b) $\ln\frac{2048}{1323}$