Day 250 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 7 m s^{-1}. 7 s later car B leaves P from rest, in the same direction, with acceleration 1.5 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 7) seconds.
B: ½×1.5×s^2 = 0.75 s^2 A: 7(s + 7)
0.75 s^2 − 7 s − 49 = 0 so s = 14 (s > 0)
(b) PQ = 0.75×14^2 = 147
(c) v = 0 + at = 1.5×14 = 21
Answer: (a) 14 s (b) 147 m (c) 21 m s^{-1}

2 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -2 \end{pmatrix}$ and $\begin{pmatrix} k \\ 1 \\ 1 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + k - 2 = 0$
(k − 1)(k + 2) = 0
k = −2 or k = 1. For k = 1 the first vector is $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, with length $\sqrt{6} = \sqrt{6}$

Answer: k = −2 or 1; length $\sqrt{6}$

3 Arithmetic modelling (All boards)

A charity raises £600 in its first year and in each later year raises £50 more than in the previous year.
(a) How much does it raise in year 15? (b) In which year does the total raised first exceed £5000?

(a) $u_{15} = 600 + 14 \times 50 = 1300$

(b) $S_m = \frac{m}{2}(1200 + (m - 1) \times 50) > 5000$

$S_6 = 4350 < 5000$ and $S_7 = 5250 > 5000$

Answer: (a) £1300 (b) year 7

4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 2 kg is acted on by forces $F_1 = (-18\mathbf{i} + 11\mathbf{j})$ N and F_2. Its acceleration is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and **i**, to 1 d.p.

F = ma: $F_1 + F_2 = 2(3\mathbf{i} + 2\mathbf{j}) = 6\mathbf{i} + 4\mathbf{j}$
(a) $F_2 = (6\mathbf{i} + 4\mathbf{j}) - (-18\mathbf{i} + 11\mathbf{j}) = 24\mathbf{i} - 7\mathbf{j}$
(b) $|F_2| = \sqrt{(24^2 + -7^2)} = 25$
cos θ = 24 ÷ 25, so θ = 16.3°
Answer: (a) $F_2 = (24\mathbf{i} - 7\mathbf{j})$ N (b) 16.3°

5 Line meets curve: length of chord (All boards)

The line y = 3x − 3 meets the curve $y = x^2 + 2x - 15$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 + 2x - 15 = 3x - 3$ gives $x^2 - x - 12 = 0$
(x + 3)(x − 4) = 0: x = −3 or 4
A(−3, −12), B(4, 9)
$AB^2 = 7^2 + 21^2 = 490$
Answer: A(−3, −12), B(4, 9); AB = $7\sqrt{10}$

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6 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 5$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$,

and use it with $x_0 = \frac{3}{2}$ to find x_2 as an approximation to $\sqrt{5}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n} = \frac{1}{2}(x_n + \frac{5}{x_n})$

$x_1 = \frac{1}{2}(\frac{3}{2} + \frac{5}{\frac{3}{2}}) = \frac{29}{12}$

$x_2 = 2.2428$

Answer: $x_2 = 2.2428$

7 Poisson over different intervals (CIE)

Faults are reported at random at an average of 6 per hour.
Find the probability that in a 20-minute period there are (a) zero events (b) at least 1.
(c) Find the probability that at least one occurs in each of two separate 20-minute periods.

$\lambda = 6 \times \frac{20}{60} = 2$, $X \sim Po(2)$

(a) $P(X = 0) = e^{-2} = 0.135$
(b) $P(X \geq 1) = 1 - P(X \leq 0) = 0.865$
(c) $(1 - 0.135)^2 = 0.748$
Answer: (a) 0.135 (b) 0.865 (c) 0.748

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 - 2x)^{-1}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(2 - 2x)^{-1} = 2^{-1}(1 + \frac{-2x}{2})^{-1}$

$2^{-1} = \frac{1}{2}$

Expand $(1 + y)^{-1}$ with $y = \frac{-2x}{2}$

Valid for $|\frac{-2x}{2}| < 1$, i.e. $|x| < 1$

Answer: $\frac{1}{2} + \frac{1}{2}x + \frac{1}{2}x^2$; $|x| < 1$

9 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 8 pairs of (x, y): $\Sigma x = 132$, $\Sigma y = 467$,
$\Sigma x^2 = 2484$, $\Sigma y^2 = 30407$, $\Sigma xy = 8651$.
Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 2484 - \frac{132^2}{8} = 306$

$S_{yy} = 30407 - \frac{467^2}{8} = 3150$

$S_{xy} = 8651 - \frac{132 \times 467}{8} = 946$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.964$

This is strong positive correlation
Answer: r = 0.964, positive correlation

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10 Fixed point iteration (All boards)

The equation $x^2 = x + 9$ has a root α. Use the iteration $x_{n+1} = \sqrt{x + 9}$ with $x_0 = 2$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt{2 + 9} = 3.3166$

$x_2 = 3.5095$, $x_3 = 3.5369$

Continuing, the values settle at 3.5414, so $\alpha = 3.54$

Answer: $x_1 = 3.3166$, $x_2 = 3.5095$, $x_3 = 3.5369$; $\alpha = 3.54$