Day 248 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Perpendicular bisector and area (All boards)

A is (−4, 2) and B is (6, −2).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (1, 0); gradient AB = $-\frac{2}{5}$

Perpendicular gradient = $\frac{5}{2}$

Equation: 5x − 2y = 5
y = 0: P = (1, 0)
Area = ½ |(6 − (−4))(0 − 2) − (1 − (−4))(−2 − 2)|
Answer: (a) 5x − 2y = 5 (b) 0

2 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 3 \\ -10 \end{pmatrix}$ are perpendicular.
Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 3k - 10 = 0$
(k − 2)(k + 5) = 0

k = −5 or k = 2. For k = 2 the first vector is $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, with length $\sqrt{9} = 3$

Answer: k = −5 or 2; length 3

3 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-5\mathbf{i} + 8\mathbf{j})$ N, $F_2 = (-24\mathbf{i} + 7\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 4 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: −5 + (−24) + a = 0, so a = 29
j: 8 + 7 + b = 0, so b = −15
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-24^2 + 7^2)} = 25$ N
a = 25 ÷ 4
Answer: (a) a = 29, b = −15 (b) 6.25 m s^{-2}

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.
Find the position vector of P and the exact distance OP.

$\mathrm{OP} = \mathrm{OA} + \frac{1}{3}\mathrm{AB} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$\mathrm{OP} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13} = \sqrt{13}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{-3} \end{pmatrix}$**, OP =** $\sqrt{\mathbf{13}}$

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5 Modulus equations (All boards)

Solve $|x + 8| = -2x + 4$.

Case 1: $x + 8 = -2x + 4$ gives $x = -\frac{4}{3}$

Case 2: $x + 8 = -(-2x + 4)$ gives $x = 12$

Check $x = -\frac{4}{3}$: right-hand side = $\frac{20}{3}$ (valid)

Check $x = 12$: right-hand side $= -20 < 0$, so reject

Answer: $x = -\frac{4}{3}$

6 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is $N(65, 5^2)$. An egg is classed as 'special' if its mass is less than 59.
10 eggs are chosen at random. Find the probability that exactly 4 are special (3 s.f.).

$P(\text{special}) = P(X < 59) = P(Z < -1.20) = 0.115$

$Y \sim B(10, 0.115)$

$P(Y = 4) = C(10, 4)(0.115)^4(0.885)^6 = 0.0177$

Answer: 0.0177

7 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 2) = -3 - 4\sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$x = \frac{-3 - 4\sqrt{6}}{\sqrt{6} - 2}$

Multiply top and bottom by $\sqrt{6} + 2$.

Numerator: $(-3 - 4\sqrt{6})(\sqrt{6} + 2) = -30 - 11\sqrt{6}$

Denominator: $6 - 4 = 2$

Answer: $x = -15 - \frac{11}{2}\sqrt{6}$

8 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The radius of a sphere increases at 2 cm s^{-1}.
Find the rate of increase of its volume when r = 2 cm, in terms of π.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi \times 2^2 \times 2 = 32\pi$

Answer: 32π cm^3/s

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(22, p)$. H_0: $p = 0.5$ is tested against H_1: $p < 0.5$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.35$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 6$

(b) Type I error = P(reject H_0 | $p = 0.5$) = 0.0262

(c) Type II error = P(not in critical region | $p = 0.35$)

$P(X \geq 7)$ when $p = 0.35$: 0.698

Answer: (a) $X \leq 6$ (b) 0.0262 (c) 0.698

10 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, $\mu = 0.25$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, weight W = 19.6 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 4.90$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 17.7$

Answer: least 4.9 N, greatest 17.7 N