Day 247 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Algebraic fractions (All boards)

Simplify $\frac{x^2 - 1}{(x - 1)} \div \frac{(x + 1)}{(x - 2)}$.

$\frac{x^2 - 1}{(x - 1)} \times \frac{(x - 2)}{(x + 1)}$

$x^2 - 1 = (x - 1)(x + 1)$

Cancel $(x + 1)$.

Answer: $\mathbf{\frac{(x - 1)(x - 2)}{(x - 1)}}$

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (\sin 2x + \cos x)\, dx$.

Integrate: $[-\frac{1}{2}\cos 2x + \sin x]$

Substitute the limits and subtract: 2

Answer: 2

3 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 16 and variance 8.

(a) Find n and p.

(b) Find P(X = 14) to 3 significant figures.

(a) $np = 16$ and $npq = 8$

$q = \frac{npq}{np} = \frac{8}{16} = 0.5$, so $p = 0.5$

$n = \frac{16}{0.5} = 32$

(b) $P(X = 14) = C(32, 14)(0.5)^{14}(0.5)^{18} = 0.110$

Answer: (a) n = 32, p = 0.5 (b) 0.110

4 Power, driving force and resistance (CIE)

A 2000 kg car has engine power 60 kW and constant resistance 1200 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 12 m s^{-1} up a hill where $\sin \alpha = \frac{1}{20}$. Take g = 10.

(a) Steady speed: driving force = resistance. $P = Fv$: $60000 = 1200\ v$

(b) Driving force $F = P \div v = 60000 \div 12 = 5000$ N

Up the hill: $F - 1200 - 2000\ g \sin \alpha = 2000\ a$

$5000 - 1200 - 1000 = 2000\ a$

$a = 1.4$

Answer: (a) 50 m s^{-1} (b) 1.4 m s^{-2}

5 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^3 in the expansion of

$(2 + x)(1 - x)^7$.

In $(1 - x)^7$: coefficient of x^2 is 21, of x^3 is -35

Coefficient $= 2 \times (-35) + 1 \times 21$

Answer: −49

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6 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -1 \end{pmatrix}$ and $\begin{pmatrix} k \\ -1 \\ 20 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - k - 20 = 0$

$(k - 5)(k + 4) = 0$

$k = -4$ or $k = 5$. For $k = 5$ the first vector is $\begin{pmatrix} 5 \\ 5 \\ -1 \end{pmatrix}$, with length $\sqrt{51} = \sqrt{51}$

Answer: $k = -4$ or 5; length $\sqrt{51}$

7 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−2, 0) and passes through B(−3, 1).

Find the equation of the circle.

Touches the x-axis at A, so the centre is (−2, r) and the radius is r.

B on the circle: $(-1)^2 + (1 - r)^2 = r^2$

$1 + 1 - 2r = 0$ so $r = 1$

Answer: $(x + 2)^2 + (y - 1)^2 = 1$

8 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan \alpha = \frac{4}{3}$, and passes through A, 12 m horizontally from O and 6 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $12 = U \cos \alpha \times t$, so $t = 12 \div (0.6\ U)$

Vertical: $6 = U \sin \alpha \times t - 4.9t^2$

Eliminate t: $y = x \tan \alpha - 4.9x^2 \div (U^2 \cos^2 \alpha)$

$6 = 16 - 4.9 \times 12^2 \div (0.36\ U^2)$

$U^2 = 1960 \div 10 = 196$, $U = 14$

Answer: 14 m s^{-1}

9 Product rule: stationary points (All boards)

$y = (x - 2)e^{-x}$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1\ e^{-x} - (x - 2)e^{-x} = (-x + 3)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-x + 3 = 0$ and $x = 3$

$y = e^{-3}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(3, e^{-3})$, maximum

10 Normal approximation to the binomial (All boards)

X ~ B(40, 0.6). Use a normal approximation to estimate $P(X \geq 22)$ to 3 significant figures.

$np = 24$, $npq = 9.6$, so Y ~ N(24, 9.6)

With continuity correction: P(Y > 21.5)

$z = \frac{21.5 - 24}{3.10} = -0.807$, $P(Z > -0.807) = 0.790$

Answer: 0.790