Day 246 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stationary points with trigonometric and exponential terms (All boards)

$y = 3x - 6\sin x$ for $0 \leq x \leq 2\pi$.

Find the exact coordinates of the stationary points and determine their nature.

$\frac{dy}{dx} = 3 - 6\cos x = 0$, so $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$ in the range

$\frac{d^2y}{dx^2} = 6\sin x$: sign at each point gives the nature

Answer: $(\frac{\pi}{3}, \pi - 3\sqrt{3})$ minimum; $(\frac{5\pi}{3}, 3\sqrt{3} + 5\pi)$ maximum

2 Polynomial division (All boards)

Find the quotient and the remainder when $2x^3 + 10x^2 + 8x - 15$ is divided by $(x + 3)$.

Quotient (matching leading terms in turn): $2x^2 + 4x - 4$

$(x + 3)(2x^2 + 4x - 4) = 2x^3 + 10x^2 + 8x - 12$

Remainder $= 2x^3 + 10x^2 + 8x - 15 - (2x^3 + 10x^2 + 8x - 12) = -3$

Answer: quotient $2x^2 + 4x - 4$, remainder -3

3 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + 3y^2 = 40$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 6y}$

Horizontal tangent: $4x + 2y = 0$, so $y = -2x$

Substitute: $10x^2 = 40$, so $x^2 = 4$ and $x = \pm 2$

Points: $(2, -4)$ and $(-2, 4)$

Answer: $(2, -4)$ and $(-2, 4)$

4 Independent and mutually exclusive events (All boards)

$P(A) = 0.7$ and $P(A \cup B) = 0.79$.

(a) Given that A and B are independent, find P(B).

(b) Given instead that A and B are mutually exclusive, find P(B).

(c) In the case in (a), find $P(A' \cap B)$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.79 = 0.7 + P(B)(1 - 0.7)$

$P(B) = \frac{0.79 - 0.7}{0.3} = 0.3$

(b) $P(A \cap B) = 0$, so $P(B) = 0.79 - 0.7 = 0.09$

(c) $P(A' \cap B) = P(A')P(B) = 0.3 \times 0.3 = 0.09$

Answer: (a) 0.3 (b) 0.09 (c) 0.09

5 Modulus equations (All boards)

Solve $|x + 7| = 4x - 7$.

Case 1: $x + 7 = 4x - 7$ gives $x = \frac{14}{3}$

Case 2: $x + 7 = -(4x - 7)$ gives $x = 0$

Check $x = \frac{14}{3}$: right-hand side $= \frac{35}{3}$ (valid)

Check $x = 0$: right-hand side $= -7 < 0$, so reject

Answer: $x = \frac{14}{3}$

6 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1500 kg car tows a 250 kg trailer by a light tow-bar on a horizontal road. Driving force 1075 N; resistances 400 N (car), 150 N (trailer). Find (a) the acceleration and the tow-bar tension. At 15 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1075 - 400 - 150 = 1750\,a$, so $a = 0.3$

Trailer: $T - 150 = 250\,a$, so $T = 150 + 250\times0.3 = 225$

(b) After the break the trailer decelerates at $150 \div 250 = 0.6$ m s^{-2}

$0 = 15^2 - 2\times0.6\times s$, so $s = 187.5$

Answer: (a) 0.3 m s^{-2}, 225 N (b) 187.5 m

7 Smallest number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a biased coin lands heads is 0.08 on each independent trial.
Find the least number of trials needed so that the probability of this happening at least once is greater than 0.95.

P(at least one) $= 1 - 0.92^n > 0.95$

$0.92^n < 0.05$

$n > \dfrac{\log 0.05}{\log 0.92} = 35.9$

Least integer $n = 36$

Answer: 36

8 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = \dfrac{12}{13}$, $\cos\alpha = \dfrac{5}{13}$, $\sin\beta = 0.6$, $\cos\beta = 0.8$

Horizontal: $T_1\cos\alpha = T_2\cos\beta$, so $T_2 = \dfrac{25}{52}\,T_1$

Vertical: $T_1\sin\alpha + T_2\sin\beta = 4\,g = 39.2$

$T_1\left(\dfrac{12}{13} + \dfrac{15}{52}\right) = 39.2$

$T_1 = 32.4$, $T_2 = 15.6$

Answer: $T_1 = 32.4$ N, $T_2 = 15.6$ N

9 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}-3\\-3\\-2\end{pmatrix} + t\begin{pmatrix}0\\-3\\1\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(−5, 0, −3) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix}2\\-3t-3\\t+1\end{pmatrix}$

$\mathbf{PF}\cdot\mathbf{d} = 0$ gives $10t + 10 = 0$, so $t = -1$

F = (−3, 0, −3); $\mathbf{PF} = \begin{pmatrix}2\\0\\0\end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{4} = 2$

Answer: F(−3, 0, −3), distance 2

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10 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = -1$, $y = 3$ so $a = -1$, $b = 3$

Through (2, 4): $4 = \frac{k}{3} + 3$

$k = (4 - 3)(3) = 3$

$x = 0$: $y = 6$; $y = 0$: $x = -2$

Answer: $k = 3$, $a = -1$, $b = 3$; (0, 6) and (−2, 0)