Day 245 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 1) = -1 - \sqrt{5}$, writing x in the form $p + q\sqrt{5}$.

$x = \dfrac{-1 - \sqrt{5}}{\sqrt{5} - 1}$

Multiply top and bottom by $\sqrt{5} + 1$.

Numerator: $(-1 - \sqrt{5})(\sqrt{5} + 1) = -6 - 2\sqrt{5}$

Denominator: $5 - 1 = 4$

Answer: $\mathbf{x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}}$

2 Exact values from the addition formulae (All boards)

By writing 195° as 150° + 45°, find the exact value of

sin 195°, in the form $\dfrac{\sqrt{a} \pm \sqrt{b}}{c}$.

$\sin 195° = \sin(150° + 45°) = \sin 150° \cos 45° + \cos 150° \sin 45°$

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + (-\frac{\sqrt{3}}{2}) \times \frac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

Answer: $\mathbf{\frac{\sqrt{2} - \sqrt{6}}{4}}$

3 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 24.5 m above the ground. At the same instant ball B is projected upwards at 24.5 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$

Height of B above the ground: $24.5\,t - 4.9t^2$

They meet when the distances add up to 24.5: $4.9t^2 + (24.5\,t - 4.9t^2) = 24.5$

so $24.5\,t = 24.5$, $t = 1$

Height of B = $24.5 \times 1 - 4.9 \times 1^2 = 19.6$

Answer: (a) 1 s (b) 19.6 m

4 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |3x - 2|$ and $y = x + 4$.

The graphs meet at A and B.

Find the coordinates of A and B.

$|3x - 2| = x + 4$

Solve $3x - 2 = \pm(x + 4)$.

$x = -\frac{1}{2}$ and $x = 3$ (both checked in the original equation)

$y = \frac{7}{2}$ and $y = 7$

Answer: $\mathbf{A(-\frac{1}{2}, \frac{7}{2})}$**, B(3, 7)**

5 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$\sin 3\theta + 4\cos 3\theta = 0.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos 3\theta \approx 1 - \frac{9}{2}\theta^2$

$1 \times 3\theta + 4(1 - \frac{9}{2}\theta^2) = 0.5$

Multiply by 2 and rearrange: $36\theta^2 - 6\theta - 7 = 0$

$\theta = \frac{6 \pm \sqrt{1044}}{72}$, so $\theta = 0.532$

Answer: a = 36, b = −6, c = −7; $\theta \approx 0.532$

6 Bayes' theorem in context (All boards)

3% of people have a disease. The screening test is positive for 96% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.03 \times 0.96 + 0.97 \times 0.02$
$= 0.0288 + 0.0194 = 0.0482$

(b) $P(\text{has it} \mid +) = \frac{0.03 \times 0.96}{0.0482}$

$= 0.598$

Answer: (a) 0.0482 (b) 0.598

7 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2 x - 2\cot x - 6 = 0$
for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $2\cot^2 x - 2\cot x - 4 = 0$
$\cot x = -1$ or $\cot x = 2$
$\cot x = -1$ means $\tan x = -1$: $x = 135.0°$
$\cot x = 2$ means $\tan x = \frac{1}{2}$: $x = 26.6°$

Answer: x = 26.6°, 135.0°

8 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (−9, −8). Its centre lies in the third quadrant. Find the equations of the two possible circles.

Centre (−r, −r), radius r > 0.
Through (−9, −8): $(-9 + r)^2 + (-8 + r)^2 = r^2$
$r^2 - 34r + 145 = 0$
$(r - 5)(r - 29) = 0$, so r = 5 or 29

Answer: $(x + 5)^2 + (y + 5)^2 = 25$ or $(x + 29)^2 + (y + 29)^2 = 841$

9 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8.

A: weight component along plane = 3 g sin 30°; B: 9 g sin 30°
B moves down, A up (9 sin 30° > 3 sin 30°): F = ma for the system
9 g sin 30° − 3 g sin 30° = (3 + 9)a
(a) a = 2.45
(b) For A: T − 3 g sin 30° = 3 a, so T = 22.0

Answer: (a) 2.45 m s^{-2}, B moves down (b) 22.05 N

10 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 53 students in sets A and B.

(a) Find the value of x.

(b) A student in B is chosen at random. Find P(A).

(a) Total: $(2x + 4) + (x + 5) + (3x + 5) + 3 = 53$

$6x + 17 = 53$, so $x = 6$

(b) Regions: 16, 11, 23, 3

In B: $11 + 23 = 34$, of which in A: 11

Answer: (a) x = 6 (b) $\frac{11}{34}$