Day 244 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 17 pairs of values of daily rainfall and ice cream sales, r = 0.693.
The critical value for a two-tailed test at the 1% level is 0.6055.
Test, at this level, whether there is any correlation between the two variables.

H_0: $\rho = 0$, H_1: $\rho \neq 0$
Critical value 0.6055
$|r| = 0.693 > 0.6055$
This is in the critical region, so reject H_0.
There is evidence of positive correlation.
Answer: Reject H_0: evidence of correlation

2 Partial fractions: distinct linear factors (All boards)

Express $\frac{-3x + 3}{(3x + 3)(3x + 1)}$ in partial fractions.

Let $\frac{-3x + 3}{(3x + 3)(3x + 1)} \equiv \frac{A}{(3x + 3)} + \frac{B}{(3x + 1)}$
$-3x + 3 \equiv A(3x + 1) + B(3x + 3)$
Substitute $x = -1$: $A = -3$
Substitute $x = -\frac{1}{3}$: $B = 2$
Answer: $\frac{-3}{(3x + 3)} + \frac{2}{(3x + 1)}$

3 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}0\\7\\3\end{pmatrix} + t\begin{pmatrix}-1\\3\\0\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(5, 2, 0) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix}-t - 5\\3t + 5\\3\end{pmatrix}$
PF·d = 0 gives $10t + 20 = 0$, so $t = -2$
F = (2, 1, 3); $\mathrm{PF} = \begin{pmatrix}-3\\-1\\3\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{19}$
Answer: F(2, 1, 3), distance $\sqrt{19}$

4 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 3 seconds before it hits the sea it falls 235.2 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 3)$ s the stone falls $4.9(t - 3)^2$.
$4.9t^2 - 4.9(t - 3)^2 = 235.2$
$4.9(6t - 9) = 235.2$, so $t = 9$
Height $= 4.9 \times 9^2 = 442.225$
Answer: 442.225 m

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5 Perpendicular bisector and area (All boards)

A is $(-6, -1)$ and B is $(4, -3)$.
(a) Find the equation of the perpendicular bisector of AB, in the form $ax + by = c$.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = $(-1, -2)$; gradient AB = $-\frac{1}{5}$

Perpendicular gradient = 5
Equation: $5x - y = -3$

$y = 0$: P = $(-\frac{3}{5}, 0)$

Area = $\frac{1}{2}|(4 - (-6))(0 - (-1)) - (-\frac{3}{5} - (-6))(-3 - (-1))|$

Answer: (a) $5x - y = -3$ (b) $\frac{52}{5}$

6 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 45) = 0.1587$ and $P(X > 53) = 0.1587$.
Find μ and σ.

z-values: 45: $z = -1.00$; 53: $z = 1.00$
$45 = \mu - 1.00\,\sigma$ and $53 = \mu + 1.00\,\sigma$
Subtract: $8 = 2.00\,\sigma$, so $\sigma = 4.00$
$\mu = 45 + 1.00 \times 4.00 = 49.0$
Answer: $\mu = 49$, $\sigma = 4$

7 Integration by substitution (All boards)

Use the substitution $u = 4 + \sin x$ to find the exact value of
$\int_0^{\frac{\pi}{2}} \frac{\cos x}{4 + \sin x}\,dx$.

$du = \cos x\,dx$; $x = 0$ gives $u = 4$, $x = \frac{\pi}{2}$ gives $u = 5$

$I = \int_4^5 \frac{1}{u}\,du = [\ln u]$
$= \ln 5 - \ln 4 = \ln 5 - 2\ln 2$
Answer: $\ln 5 - 2\ln 2$

8 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 48t + 90)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 6 s.

(a) $v = 6(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$
(b) Displacement $x = \int v\,dt = 2t^3 - 24t^2 + 90t$
$x(3) = 108$, $x(5) = 100$, $x(6) = 108$
Direction changes at $t = 3$ and $t = 5$, so distance $= |108| + |100 - 108| + |108 - 100|$
Answer: (a) $t = 3$, $t = 5$ (b) 124 m

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{7x - 4}{(1 - x)(1 + 2x)}$.
(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 + 2x)}$: $A = 1$, $B = -5$
(b) $1(1 - x)^{-1} = 1(1 + x + 1x^2 \ldots)$
Add the series for both fractions.
(c) Need $|x| < 1$ and $|2x| < 1$
Answer: (a) $\frac{1}{(1 - x)} + \frac{-5}{(1 + 2x)}$ (b) $-4 + 11x - 19x^2$ (c) $|x| < \frac{1}{2}$

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10 Exponential equations reducing to quadratics (All boards)

Solve $e^{x} + 27\,e^{-x} = 12$, giving exact answers.

Multiply by e^{x}: $e^{2x} + 27 = 12\,e^{x}$

Let $u = e^{x}$: $u^{2} - 12u + 27 = 0$, so $(u - 3)(u - 9) = 0$

$e^{x} = 3$ or $e^{x} = 9$

Answer: $x = \ln 3$ or $x = 2 \ln 3$