Day 243 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "If ab is a multiple of 12, then a or b is a multiple of 12" is false.

Take a = 2 and b = 6:
ab = 12 is a multiple of 12,
but neither 2 nor 6 is a multiple of 12.
Answer: a = 2, b = 6

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m s^{-1} and accelerates uniformly. She travels 91.2 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $91.2 = 4\times12 + \frac{1}{2}\times a\times12^2$, so $a = 0.6$
(b) $v = u + at = 4 + 0.6\times12 = 11.2$
(c) For the midpoint, $s = 45.6$: $v^2 = u^2 + 2as = 4^2 + 2\times0.6\times45.6$
$v^2 = 70.72$
Answer: (a) 0.6 m s^{-2} (b) 11.2 m s^{-1} (c) 8.41 m s^{-1}

3 Expectation and variance of linear combinations (Edexcel, OCR, MEI, Eduqas, CIE)

X has E(X) = 4 and Var(X) = 1.
(a) Find $E(X^2)$.
(b) $Y = -2X + 9$. Find E(Y) and Var(Y).
(c) X_1 and X_2 are independent copies of X. Find $Var(X_1 - X_2)$.

(a) $E(X^2) = Var(X) + [E(X)]^2 = 1 + 16 = 17$
(b) $E(Y) = -2 \times 4 + 9 = 1$, $Var(Y) = (-2)^2 \times 1 = 4$
(c) $Var(X_1 - X_2) = Var(X_1) + Var(X_2) = 2 \times 1 = 2$
Answer: (a) 17 (b) E(Y) = 1, Var(Y) = 4 (c) 2

4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $F_1 = (-16\mathbf{i} + 5\mathbf{j})$ N and F_2. Its acceleration is $(2\mathbf{i} + 3\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

F = ma: $F_1 + F_2 = 4(2\mathbf{i} + 3\mathbf{j}) = 8\mathbf{i} + 12\mathbf{j}$
(a) $F_2 = (8\mathbf{i} + 12\mathbf{j}) - (-16\mathbf{i} + 5\mathbf{j}) = 24\mathbf{i} + 7\mathbf{j}$
(b) $|F_2| = \sqrt{(24^2 + 7^2)} = 25$
$\cos\theta = 24 \div 25$, so $\theta = 16.3°$
Answer: (a) $F_2 = (24\mathbf{i} + 7\mathbf{j})$ N (b) 16.3°

5 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = e^{2x}$, the x-axis, x = 0 and x = 1 is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = e^{4x}$, so $V = \pi\int_0^1 y^2\,dx$

$= \pi \times \frac{e^4 - 1}{4}$

$V = \frac{\pi e^4 - \pi}{4}$

Answer: $\frac{\pi e^4 - \pi}{4}$

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6 Perpendicular bisector and area (All boards)

A is (−3, 1) and B is (1, 3).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (−1, 2); gradient AB = $\frac{1}{2}$
Perpendicular gradient = −2
Equation: 2x + y = 0
y = 0: P = (0, 0)
Area = ½ |(1 − (−3))(0 − 1) − (0 − (−3))(3 − 1)|
Answer: (a) 2x + y = 0 (b) 5

7 Exponential equations reducing to quadratics (All boards)

Solve $e^x + 18\,e^{-x} = 11$, giving exact answers.

Multiply by e^x: $e^{2x} + 18 = 11\,e^x$
Let $u = e^x$: $u^2 - 11u + 18 = 0$, so $(u - 2)(u - 9) = 0$
$e^x = 2$ or $e^x = 9$
Answer: $x = \ln 2$ or $x = 2 \ln 3$

8 Normal approximation to the Poisson (CIE)

X ~ Po(18). Use a suitable approximation to find P(X ≤ 13), to 3 significant figures.

λ > 15, so Y ~ N(18, 18) approximately
Continuity correction: P(Y < 13.5)
z = −1.06, so probability = 0.144
Answer: 0.144

9 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (−12, −6). Its centre lies in the third quadrant. Find the equations of the two possible circles.

Centre (−r, −r), radius r > 0.
Through (−12, −6): $(-12 + r)^2 + (-6 + r)^2 = r^2$
$r^2 - 36r + 180 = 0$
(r − 6)(r − 30) = 0, so r = 6 or 30
Answer: $(x + 6)^2 + (y + 6)^2 = 36$ or $(x + 30)^2 + (y + 30)^2 = 900$

10 Product rule: stationary points (All boards)

$y = (3x + 3)e^{-x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 3\,e^{-x} - (3x + 3)e^{-x} = (-3x)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-3x = 0$ and $x = 0$

$y = 3$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: (0, 3), maximum