Day 242 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 80 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $80 = 6\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.4$
(b) $v = u + at = 6 + 0.4\times10 = 10$
(c) For the midpoint, $s = 40$: $v^2 = u^2 + 2as = 6^2 + 2\times0.4\times40$
$v^2 = 68$

Answer: (a) 0.4 m s^{-2} (b) 10 m s^{-1} (c) 8.25 m s^{-1}

2 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (−3, 2).
Find the coordinates of the corresponding point on:
(a) y = 2f(x)
(b) y = −f(x)
(c) y = f(−x)

(a) y = 2f(x): (−3, 4)
(b) y = −f(x): (−3, −2)
(c) y = f(−x): (3, 2)
A stretch or translation moves the point in the stated direction only.

Answer: (a) (−3, 4); (b) (−3, −2); (c) (3, 2)

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 585 items.
The 6th item chosen is number 80 and the 8th item chosen is number 110.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = (110 − 80) ÷ (8 − 6) = 15
First item = 80 − 5 × 15 = 5
Sample size = 585 ÷ 15 = 39

Answer: Interval 15, first 5, size 39

4 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = 5
Line AB: y − 3 = 0(x − −3). Solve to get D = (5, 3)
Area = ½ × AB × CD = 27

Answer: (a) x = 5 (b) D(5, 3), area 27

5 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 6 m s^{-1} from a point 19.6 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $19.6 = \frac{1}{2}\times9.8\times t^2$, so $t = 2$
(a) Horizontal distance = 6 × 2
(b) Vertical speed on landing = 9.8×2 = 19.6; horizontal speed = 6
Speed = $\sqrt{(6^2 + 19.6^2)} = 20.5$
$\tan\theta = 19.6 \div 6$, $\theta = 73.0°$

Answer: (a) 12 m (b) 20.5 m s^{-1} at 73.0° below the horizontal

6 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-2x^2 - xy + 3y^2 + 4y = 16$.
Show that P(−2, 2) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(-4x - y) + (-x + 6y + 4)\frac{dy}{dx} = 0$

At P(−2, 2): $6 + (18)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{1}{3}$

Tangent: $y - 2 = (-\frac{1}{3})(x + 2)$, i.e. $x + 3y - 4 = 0$

Answer: $x + 3y - 4 = 0$

7 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 1) = -1 + 2\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$x = \frac{-1 + 2\sqrt{2}}{\sqrt{2} - 1}$

Multiply top and bottom by $\sqrt{2} + 1$.
Numerator: $(-1 + 2\sqrt{2})(\sqrt{2} + 1) = 3 + \sqrt{2}$
Denominator: $2 - 1 = 1$
Answer: $x = 3 + \sqrt{2}$

8 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 13 \\ -3 \\ -1 \end{pmatrix} + t\begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$.
Find the coordinates of the foot of the perpendicular from P(4, −2, 2) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} -3t + 9 \\ -1 \\ -3 \end{pmatrix}$
$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $9t - 27 = 0$, so $t = 3$
F = (4, −3, −1); $\mathbf{PF} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{10}$
Answer: F(4, −3, −1), distance $\sqrt{10}$

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + 3y^2 = 10$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 6y}$
Horizontal tangent: $4x + 2y = 0$, so $y = -2x$
Substitute: $10x^2 = 10$, so $x^2 = 1$ and $x = \pm 1$
Points: (1, −2) and (−1, 2)
Answer: (1, −2) and (−1, 2)

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(22, p). H_0: p = 0.5 is tested against H_1: p < 0.5 using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when p = 0.4 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 6$
(b) Type I error = P(reject H_0 | p = 0.5) = 0.0262
(c) Type II error = P(not in critical region | p = 0.4)
$P(X \geq 7)$ when p = 0.4: 0.842
Answer: (a) $X \leq 6$ (b) 0.0262 (c) 0.842