**Day 241** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Chain rule and gradients (All boards)

$y = (\ln x)^3$ for $x > 0$.

Find $\frac{dy}{dx}$ and its exact value at $x = e$.

Let $u = \ln x$: $y = u^3$, so $\frac{dy}{dx} = 3u^2 \times \frac{1}{x} = \frac{3(\ln x)^2}{x}$

At $x = e$: $\ln x = 1$, so gradient $= \frac{3}{e}$

**Answer:** $\mathbf{\frac{dy}{dx} = \frac{3(\ln x)^2}{x}}$**; gradient** $\mathbf{= \frac{3}{e}}$

## 2 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take $g = 9.8$.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$; friction $F = \mu R = 0.25 \times 6\,g\cos\alpha$
B: $12g - T = 12a$
A: $T - 6g\sin\alpha - F = 6a$
Adding: $g(12 - 3.6 - 1.2) = 18a$
(a) $a = 3.92$; (b) $T = 12(g - a)$
**Answer: (a) 3.92 m s$^{-2}$ (b) 70.56 N**

## 3 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a component is faulty is 0.25. In 21 independent trials, find the probability that this happens (a) exactly 7 times (b) fewer than 7 times (c) at least 7 times.

Let $X \sim B(21, 0.25)$
(a) $P(X = 7) = 0.126$
(b) $P(X \leq 6) = 0.744$
(c) $1 - P(X \leq 6) = 0.256$
**Answer: (a) 0.126 (b) 0.744 (c) 0.256**

## 4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $\mathbf{F}_1 = (-2\mathbf{i} + 18\mathbf{j})$ N and $\mathbf{F}_2$. Its acceleration is $(2\mathbf{i} + 4\mathbf{j})$ m s$^{-2}$. Find (a) $\mathbf{F}_2$, (b) the angle between $\mathbf{F}_2$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F}_1 + \mathbf{F}_2 = 3(2\mathbf{i} + 4\mathbf{j}) = 6\mathbf{i} + 12\mathbf{j}$
(a) $\mathbf{F}_2 = (6\mathbf{i} + 12\mathbf{j}) - (-2\mathbf{i} + 18\mathbf{j}) = 8\mathbf{i} - 6\mathbf{j}$
(b) $|\mathbf{F}_2| = \sqrt{(8^2 + -6^2)} = 10$
$\cos\theta = 8 \div 10$, so $\theta = 36.9°$
**Answer: (a) $\mathbf{F}_2 = (8\mathbf{i} - 6\mathbf{j})$ N (b) 36.9°**

## 5 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

$n = 8$ pairs of $(x, y)$: $\Sigma x = 168$, $\Sigma y = 384$, $\Sigma x^2 = 4128$, $\Sigma xy = 8912$.
(a) Find the equation of the regression line of $y$ on $x$ (3 s.f.).
(b) Estimate $y$ when $x = 9$.

$S_{xx} = 4128 - \frac{168^2}{8} = 600$, $S_{xy} = 8912 - \frac{168 \times 384}{8} = 848$

$b = \frac{S_{xy}}{S_{xx}} = 1.41$
$a = \bar{y} - b\,\bar{x} = 48.0 - (1.41)(21.0) = 18.3$
(b) $y = 18.3 + (1.41)(9) = 31.0$
**Answer: (a) $y = 18.3 + 1.41x$ (b) 31.0**

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## 6 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \times 2^{2x} - 6 \times 2^x + 4 = 0$.

Let $y = 2^x$. Then $2y^2 - 6y + 4 = 0$

$2(y - 2)(y - 1) = 0$

$y = 2$ or $y = 1$

$2^x = 2$ gives $x = 1$; $2^x = 1$ gives $x = 0$

**Answer: $x = 0$ or $x = 1$**

## 7 Partial fractions and integration (All boards)

(a) Express $\frac{2x + 9}{(x + 3)^2}$ in the form $\frac{A}{x + 3} + \frac{B}{(x + 3)^2}$.

(b) Hence find the exact value of $\int_0^3 \frac{2x + 9}{(x + 3)^2}\,dx$.

$2x + 9 \equiv A(x + 3) + B$, so $A = 2$ and $B = 3$

$I = [2\ln(x + 3) - \frac{3}{x + 3}]_0^3$

$= 2\ln\frac{6}{3} + 3(\frac{1}{3} - \frac{1}{6})$

$= \frac{4\ln 2 + 1}{2}$

**Answer: (a) $A = 2$, $B = 3$; (b) $\frac{4\ln 2 + 1}{2}$**

## 8 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(2, −9) and B(14, −1).

Its centre lies on the line $y = \frac{1}{2}x + 3$.

Find the equation of the circle.

Centre $(h, \frac{1}{2}h + 3)$ is equidistant from A and B.

$(h - 2)^2 + (\frac{1}{2}h + 12)^2 = (h - 14)^2 + (\frac{1}{2}h + 4)^2$

Solving gives $h = 2$, so centre (2, 4)

$r^2 = (0)^2 + (-13)^2 = 169$

**Answer: $(x - 2)^2 + (y - 4)^2 = 169$**

## 9 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(0, 4, 3) to l, and the exact distance from P to l.

Foot $F = A + t\,d$: $\mathbf{PF} = \begin{pmatrix} 3t + 4 \\ -t - 5 \\ 3t + 7 \end{pmatrix}$

$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $19t + 38 = 0$, so $t = -2$

$F = (-2, 1, 4)$; $\mathbf{PF} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{14}$

**Answer: F(−2, 1, 4), distance $\sqrt{14}$**

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## 10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-5x}{(1 + 3x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to $x^2$.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 3x)} + \frac{B}{(1 + 2x)}$: A = 5, B = −5

(b) $5(1 + 3x)^{-1} = 5(1 - 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need |3x| < 1 and |2x| < 1

**Answer: (a)** $\frac{5}{(1 + 3x)} + \frac{-5}{(1 + 2x)}$ **(b)** $-5x + 25x^2$ **(c)** $|x| < \frac{1}{3}$