Day 240 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(14, 0.3). Find, to 3 significant figures:
(a) P(X = 3) (b) P(X ≥ 3) (c) P(X ≤ 2)

(a) $C(14, 3)(0.3)^3(0.7)^{11} = 0.194$
(b) $1 - P(X \leq 2) = 1 - 0.161 = 0.839$
(c) $P(X \leq 2) = 0.161$
Answer: (a) 0.194 (b) 0.839 (c) 0.161

2 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 4x + 4y - 17 = 0$ passes through P(−5, −6).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−2, −2). Gradient of radius CP = $\frac{4}{3}$

Gradient of tangent = $-\frac{3}{4}$

$y + 6 = -\frac{3}{4}(x + 5)$: $3x + 4y = -39$

y = 0: x = −13
Answer: (a) 3x + 4y = −39 (b) (−13, 0)

3 Change of sign to locate a root (All boards)

$f(x) = x^3 - 3x - 4$.
Show that the equation f(x) = 0 has a root α in the interval [2, 3].

f(x) is continuous on the interval
f(2) = −2.00 < 0
f(3) = 14.0 > 0
A change of sign, so a root lies in [2, 3]
Answer: Sign change: f(2) = −2.00, f(3) = 14.0

4 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 3750 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 100) × V
3750 = 125 V, so V = 30
(b) Deceleration = V ÷ 25 = 30 ÷ 25
Answer: (a) V = 30 (b) 1.2 m s^{-2}

5 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + \sin x - 2 = 0$
for 0° ≤ x ≤ 180°.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - \sin x = 0$

Solving the quadratic: sin x = 0 or $\sin x = \frac{1}{2}$

sin x = 0: x = 0°, 180°

$\sin x = \frac{1}{2}$: x = 30°, 150°

Answer: x = 0°, 30°, 150°, 180°

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6 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 15 kg box is pulled from rest along rough horizontal ground by a horizontal force of 66.3 N and moves 25 m in 10 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $25 = \frac{1}{2}a \times 10^2$, so $a = 0.5$
(a) $P - \mu 15\,g = 15\,a$: $66.3 - 147\,\mu = 7.5$, so $\mu = 0.4$
(b) $v = at = 0.5 \times 10 = 5$; after removal deceleration $= \mu g = 3.92$
$0 = 5^2 - 2 \times 3.92 \times s$, $s = 3.19$
Answer: (a) $\mu = 0.4$ (b) 3.19 m

7 Modulus inequalities (All boards)

Solve $|4x - 5| > |2x - 3|$.

Square both sides: $(4x - 5)^2 > (2x - 3)^2$
$((4x - 5) - (2x - 3))((4x - 5) + (2x - 3)) > 0$
$(2x - 2)(6x - 8) > 0$

Critical values $x = 1$ and $x = \frac{4}{3}$; sketch the (positive or negative) quadratic.

Answer: $x < 1$ or $x > \frac{4}{3}$

8 Conditional probability and independence (All boards)

P(B) = 0.35, P(A) = 0.4 and P(A | B) = 0.4.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.4 \times 0.35 = 0.14$
$P(A \cup B) = 0.4 + 0.35 - 0.14 = 0.61$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.14}{0.4} = 0.35$
(c) $P(A)P(B) = 0.4 \times 0.35 = 0.14$, which equals $P(A \cap B) = 0.14$
So A and B are independent
Answer: (a) 0.14, 0.61 (b) 0.35 (c) independent

9 Inverse of a quadratic function (All boards)

$f(x) = 2x^2 + 4x - 1$, $x \geq -1$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 2(x + 1)^2 - 3$
Let $y = f(x)$: $(x + 1)^2 = \frac{y + 3}{2}$
$x \geq -1$, so take the positive root: $x = -1 + \sqrt{\frac{y + 3}{2}}$
Answer: $f^{-1}(x) = -1 + \sqrt{\frac{x + 3}{2}}$, $x \geq -3$

10 Differentiating sec, cosec, tan and cot (All boards)

$y = x \tan(x)$.

Find the exact gradient at $x = \frac{\pi}{6}$.

Product rule: $\frac{dy}{dx} = \tan(x) + x(\sec^2(x))$
At $x = \frac{\pi}{6}$: $\tan(x) = \frac{\sqrt{3}}{3}$ and the derivative term $= \frac{4}{3}$
Gradient $= \frac{\sqrt{3}}{3} + \frac{\pi}{6} \times \frac{4}{3} = \frac{3\sqrt{3} + 2\pi}{9}$
Answer: gradient $\frac{3\sqrt{3} + 2\pi}{9}$