Day 239 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}1\\1\\0\end{pmatrix}$, $\begin{pmatrix}1\\3\\0\end{pmatrix}$ and $\begin{pmatrix}1\\k\\0\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$\text{AB} = \begin{pmatrix}0\\2\\0\end{pmatrix}$, and $\text{AC} = \begin{pmatrix}0\\k-1\\0\end{pmatrix}$

AC = λAB: component 2 gives $5 = 2\lambda$, so $\lambda = \frac{5}{2}$

Hence $k = 1 + \frac{5}{2} \times 2 = 6$

$\text{AC} = \frac{5}{2}$ AB, so $\text{BC} = \frac{3}{2}$ AB: AB : BC = 2 : 3

Answer: k = 6, AB : BC = 2 : 3

2 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last second before it hits the sea it falls 83.3 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 1)$ s the stone falls $4.9(t - 1)^2$.
$4.9t^2 - 4.9(t - 1)^2 = 83.3$
$4.9(2t - 1) = 83.3$, so $t = 9$
Height $= 4.9 \times 9^2 = 396.9$
Answer: 396.9 m

3 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a seed germinates is 0.1. In a sample of 18, the number of successes is 0. Test at the 10% level whether p has decreased.

H_0: p = 0.1, H_1: p < 0.1; X ~ B(18, 0.1)
$P(X \leq 0) = 0.150$
0.150 > 0.1
Do not reject H_0: insufficient evidence of a change in p.
Answer: p-value 0.150; do not reject H_0

4 Conservation of momentum (CIE)

Particles A (6 kg) and B (8 kg) move towards each other on a smooth horizontal line at 8 m s^{-1} and 3 m s^{-1}. They collide directly, and A then moves in the opposite direction at 3 m s^{-1}. Find (a) the speed of B after the collision, (b) the magnitude of the impulse of A on B.

Take the initial direction of A as positive. Conservation of momentum:
$6 \times 8 + 8 \times (-3) = 6 \times (-3) + 8v$
$24 = -18 + 8v$, so $v = 5.25$
(b) Impulse on B = change in momentum of B $= 8(5.25 - (-3))$
$= 8 \times 8.25$
Answer: (a) 5.25 m s^{-1} (b) 66 N s

5 Cubic inequalities (All boards)

Given that $(x - 1)$ is a factor of $f(x) = x^3 - x$, solve $f(x) \leq 0$.

Factorise: $f(x) = (x + 1)x(x - 1)$
Roots at $x = -1, 0, 1$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) \leq 0$.
Answer: $x \leq -1$ or $0 \leq x \leq 1$

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6 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^1 xe^{-2x}\,dx$.

$u = x$, $\frac{dv}{dx} = e^{-2x}$: $\int xe^{-2x}\,dx = [\frac{xe^{-2x}}{-2}] - \int \frac{e^{-2x}}{-2}\,dx$

$= [\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}]$

$= \frac{1 - 3e^{-2}}{4}$

Answer: $\mathbf{\frac{1 - 3e^{-2}}{4}}$

7 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 3) = -5 - 2\sqrt{5}$, writing x in the form $p + q\sqrt{5}$.

$x = \frac{-5 - 2\sqrt{5}}{\sqrt{5} - 3}$

Multiply top and bottom by $\sqrt{5} + 3$.

Numerator: $(-5 - 2\sqrt{5})(\sqrt{5} + 3) = -25 - 11\sqrt{5}$

Denominator: $5 - 9 = -4$

Answer: $\mathbf{x = \frac{25}{4} + \frac{11}{4}\sqrt{5}}$

8 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 4(t - \sin t)$, $y = 4(1 - \cos t)$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{3}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{\sin t}{1 - \cos t}$

$t = \frac{\pi}{3}$: gradient $m = \sqrt{3}$, point $(\frac{4\pi - 6\sqrt{3}}{3}, 2)$

$c = y - mx = \frac{24 - 4\sqrt{3}\pi}{3}$

Answer: $\mathbf{\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}}$; $\mathbf{y = \sqrt{3}x + \frac{24 - 4\sqrt{3}\pi}{3}}$

9 Interpolation for median and percentiles (All boards)

The table summarises the length, l mm, of leaves of 56 items.

Use linear interpolation to estimate the lower quartile.

Position = $0.25 \times 56 = 14$

Cumulative frequency before the class $30 \leq l < 40$ is 13

Estimate = $30 + \frac{14 - 13}{2} \times 10$

$= 35.0$

Answer: 35.0

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10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.02 to estimate $\sqrt{104}$ to 3 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, y = 2x

(a) $1 + x - \frac{1}{2}x^2$

$x = \frac{1}{50}$: $104 = 10^2(1 + \frac{1}{25})$ so $\sqrt{104} = 10 \times (1 + \frac{1}{25})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{50} - \frac{1}{5000}) = 10.198000$

Answer: (a) $1 + x - \frac{1}{2}x^2$ (b) 10.198