Day 238 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Heathrow: mean 11.5, standard deviation 5.9;
Leuchars: mean 22, standard deviation 4.8.
Make two comparisons.

Means: Leuchars has the higher mean (22 against 11.5), so on average the total rainfall is greater at Leuchars
Standard deviations: Heathrow is larger at Heathrow (5.9 against 4.8)
so the readings at Heathrow are more variable and those at Leuchars are more consistent
Answer: Leuchars higher on average; Heathrow more variable

2 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (−2, −4).
Find the coordinates of the corresponding point on:
(a) y = f(2x)
(b) y = f(−x)
(c) y = −f(x)

(a) y = f(2x): (−1, −4)
(b) y = f(−x): (2, −4)
(c) y = −f(x): (−2, 4)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (−1, −4); (b) (2, −4); (c) (−2, 4)

3 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that
$\frac{\sin 2\theta \tan 4\theta}{1 - \cos 4\theta}$
approaches as $\theta \to 0$.

$\sin 2\theta \approx 2\theta$, $\tan 4\theta \approx 4\theta$, $1 - \cos 4\theta \approx \frac{1}{2}(4\theta)^2$

Expression $\approx \frac{2\theta \times 4\theta}{\frac{16}{2}\theta^2} = \frac{8\theta^2}{\frac{16}{2}\theta^2} = 1$

Answer: 1

4 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last second before it hits the sea it falls 63.7 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first (t − 1) s the stone falls $4.9(t - 1)^2$.
$4.9t^2 - 4.9(t - 1)^2 = 63.7$
$4.9(2t - 1) = 63.7$, so t = 7
Height = $4.9 \times 7^2 = 240.1$
Answer: 240.1 m

5 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = $-\frac{1}{3}$, so gradient of the perpendicular = 3

Through C: 3x − y = 4

Line AB: $y - 3 = -\frac{1}{3}(x - -3)$. Solve to get D = $(\frac{9}{5}, \frac{7}{5})$

Area = ½ × AB × CD = 12

Answer: (a) 3x − y = 4 (b) D$(\frac{9}{5}, \frac{7}{5})$, area 12

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6 Product rule: stationary points (All boards)

$y = (4x + 2)e^{-2x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4\,e^{-2x} - 2(4x + 2)e^{-2x} = (-8x)e^{-2x}$

$e^{-2x} > 0$, so $\frac{dy}{dx} = 0$ gives $-8x = 0$ and $x = 0$

$y = 2$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: (0, 2), maximum

7 Bayes' theorem in context (All boards)

10% of people have a disease. The screening test is positive for 96% of those who have it and for 10% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.1 \times 0.96 + 0.9 \times 0.1$
$= 0.096 + 0.09 = 0.186$

(b) $P(\text{has it} \mid +) = \frac{0.1 \times 0.96}{0.186}$

$= 0.516$

Answer: (a) 0.186 (b) 0.516

8 Tangent conditions (All boards)

The line $y = kx + 2$ is a tangent to the curve $y = x^2 - 5x + 11$.
(a) Find the possible values of k.
(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 - 5x + 11 = kx + 2$
$x^2 + (-5 - k)x + 9 = 0$
Tangent: discriminant = 0: $(-5 - k)^2 = 36$
$-5 - k = \pm 6$ so $k = 1$ or -11
$k = 1$: $x = \frac{k - (-5)}{2} = 3$, $y = 5$

Answer: (a) $k = -11$ or $k = 1$ (b) (3, 5)

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A\,b^x$.
Find the exact values of A and b.

$\ln y = \ln A + x \ln b$, a straight line with gradient $\ln b$ and intercept $\ln A$
Intercept $\ln A = 3$, so $A = e^3$
Gradient $\ln b = \frac{7 - 3}{2 - 0} = 2$, so $b = e^2$

Answer: $A = e^3$, $b = e^2$

10 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = $7g \sin 60°$; B: $9g \sin 30°$
A moves down, B up ($7 \sin 60° > 9 \sin 30°$): F = ma for the system
$7g \sin 60° - 9g \sin 30° = (7 + 9)a$
(a) $a = 0.957$
(b) For A: $7g \sin 60° - T = 7a$, so $T = 52.7$

Answer: (a) 0.957 m s^{-2}, A moves down (b) 52.7 N