**Day 237** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = -\sin\theta$ for $-180° \le \theta \le 180°$.

$\sin 2\theta \equiv 2\sin\theta\cos\theta$: $\sin\theta(2\cos\theta + 1) = 0$
$\sin\theta = 0$: $\theta = -180°, 0°, 180°$
$\cos\theta = -\frac{1}{2}$: $\theta = -120°, 120°$
Do not divide by $\sin\theta$, as that would lose solutions
**Answer: $\theta = -180°, -120°, 0°, 120°, 180°$**

## 2 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 104.4 m above the sea at 20 m $s^{-1}$, at angle $\alpha$ above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 16, vertical component 12 (upwards)
(a) $-104.4 = 12t - 4.9t^2$, so $4.9t^2 - 12t - 104.4 = 0$
$t = [12 + \sqrt{2190.24}] \div 9.8 = 6$ (taking the positive root)
(b) Distance = 16×6
(c) Vertical speed = $12 - 9.8\times6 = -46.8$; speed = $\sqrt{(16^2 + 46.8^2)}$
**Answer: (a) 6 s (b) 96 m (c) 49.5 m $s^{-1}$**

## 3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^4 (2\sqrt{x} + \frac{5}{\sqrt{x}})\,dx$.

Integrate: $[\frac{4}{3}x^{\frac{3}{2}} + 10x^{\frac{1}{2}}]$

Substitute the limits and subtract: $\frac{58}{3}$

**Answer: $\frac{58}{3}$**

## 4 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (4 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $6g - T = 6a$; B: $T - 4g = 4a$
$a = 2g \div 10 = 1.96$, so $T = 4(g + a) = 47.04$
When A lands, B has risen 2 m with $v^2 = 2\times1.96\times2 = 7.84$
B then moves freely under gravity: extra height = $v^2 \div (2\times9.8) = 0.4$
Total = 2 + 0.4
**Answer: (a) 47.04 N (b) 2.4 m**

## 5 Equations with sec x and tan x (All boards)

Solve $\sec^2 x - \tan x - 3 = 0$
for $0° \le x \le 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $\tan^2 x - \tan x - 2 = 0$
$\tan x = 2$ or $\tan x = -1$
$\tan x = 2$: $x = 63.4°$
$\tan x = -1$: $x = 135.0°$
**Answer: x = 63.4°, 135.0°**

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## 6 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{4}{x-3} - 2$, is translated by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = 3, y = −2

Translate by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$: x = 5, y = −4

New curve: y = $\frac{4}{x-5} - 4$

y = 0: $\frac{4}{x-5} = 4$, so x − 5 = 1

**Answer: x = 5, y = −4; crosses x-axis at (6, 0)**

## 7 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the journey time (minutes), is N(29, $6^2$). The highest 5% of values exceed k.

Find k to 3 significant figures.

P(X > k) = 0.05, so P(X < k) = 0.95

$z = \Phi^{-1}(0.95) = 1.64$

k = 29 + 1.64 × 6 = 38.9

**Answer: 38.9**

## 8 Partial fractions and binomial expansion (All boards)

f(x) = $\frac{-12x-3}{(1-x)(1+2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to $x^2$.

(c) State the range of x for which the expansion is valid.

(a) f(x) = $\frac{A}{(1-x)} + \frac{B}{(1+2x)}$: A = −5, B = 2

(b) $-5(1-x)^{-1} = -5(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need |x| < 1 and |2x| < 1

**Answer: (a)** $\frac{-5}{(1-x)} + \frac{2}{(1+2x)}$ **(b)** $-3 - 9x + 3x^2$ **(c)** $|x| < \frac{1}{2}$

## 9 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (−2, −1). Its centre lies in the third quadrant.

Find the equations of the two possible circles.

Centre (−r, −r), radius r > 0.

Through (−2, −1): $(-2 + r)^2 + (-1 + r)^2 = r^2$

$r^2 - 6r + 5 = 0$

(r − 1)(r − 5) = 0, so r = 1 or 5

**Answer:** $(x + 1)^2 + (y + 1)^2 = 1$ **or** $(x + 5)^2 + (y + 5)^2 = 25$

## 10 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The length of a plank is N(70, $4^2$). A plank is classed as 'special' if its length is less than 63.

12 planks are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X < 63) = P(Z < −1.75) = 0.0401

Y ~ B(12, 0.0401)

$P(Y = 3) = C(12, 3)(0.0401)^3(0.960)^9 = 0.00979$

**Answer: 0.00979**