Day 236 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (10 kg, 1.5 m) is smoothly hinged at A to a vertical wall, with a 2 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{3}{4}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 1.5 = 10g \times 0.75 + 2g \times 1.5$
$T\sin\theta = (5 + 2)g = 68.6$, so (a) $T = 114$
Horizontal component of hinge force: $X = T\cos\theta = 91.5$
Vertical: $Y + T\sin\theta = (10 + 2)g$, so $Y = 10g \div 2 = 49$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(91.5^2 + 49^2)}$
Answer: (a) 114 N (b) 104 N

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(-3 - x)^7$.

Term in x^2: ${}^7C_2 \times (-3)^5 \times (-1x)^2$
$= 21 \times -243 \times 1$
Answer: -5103

3 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of NUMBERS have R and E not next to each other?

All arrangements = 7! = 5040
R and E together: $2 \times 6! = 1440$
Not together = $5040 - 1440 = 3600$
Answer: 3600

4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + n + 7$ is odd for every positive integer n.

$n^2 + n + 7 = n(n + 1) + 7$
n(n + 1) is the product of consecutive integers, so it is even.
Even + 7 (odd) = odd.
Answer: n(n + 1) is even, so $n^2 + n + 7$ is odd

5 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve $y = x(x - 4)(x - 5)$.
Find the total area of the two shaded regions.

$y = x^3 - 9x^2 + 20x$; $\int y\,dx = \frac{1}{4}x^4 - 3x^3 + 10x^2$

From 0 to 4: 32 (above the axis)

From 4 to 5: $-\frac{3}{4}$ (below the axis), so the area is $\frac{3}{4}$

Total $= 32 + \frac{3}{4} = \frac{131}{4}$

Answer: $\frac{131}{4}$

6 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 30 items.
Use linear interpolation to estimate the upper quartile.

Position $= 0.75 \times 30 = 22.5$
Cumulative frequency before the class $80 \leq t < 90$ is 22

Estimate $= 80 + \frac{22.5 - 22}{6} \times 10$

$= 80.8$
Answer: 80.8

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7 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 15 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m. A 5 kg particle is placed at E, AE = x m, and the plank is about to tilt about D. Find x.

About to tilt about D, so the reaction at C is zero and E lies between D and B.
Moments about D: $5\,g \times (x - 2.5) = 15\,g \times (2.5 - 2)$
$5(x - 2.5) = 7.5$
$x = 2.5 + 1.5$
Answer: x = 4

8 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 25, and after 6 hours h = 4.
Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + C$

$t = 0$: $C = 2 \times 5 = 10$, so $2\sqrt{h} = 10 - kt$
$t = 6$, $h = 4$: $4 = 10 - 6k$, so $k = 1$

Empty when $h = 0$: $kt = 10$, so $t = \frac{10}{1} = 10$ hours

Answer: 10 hours

9 Exponential growth and decay modelling (All boards)

The population of a town is modelled by $P = 100\,e^{kt}$, where t is time in years.
After 5 years, P = 500.
Find (a) the exact value of k, (b) the time for P to reach 1000, to 3 s.f.

$100\,e^{5k} = 500$, so $e^{5k} = 5$ and $k = \frac{\ln 5}{5}$

$k = 0.322$ (3 s.f.)

$100\,e^{kt} = 1000$ gives $kt = \ln 10$, so $t = \frac{\ln 10}{k} = 7.15$ years

Answer: (a) $k = \frac{\ln 5}{5}$ (b) t = 7.15 years

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-11x + 6}{(1 - x)(1 - 2x)}$.
(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 - 2x)}$: A = 5, B = 1
(b) $5(1 - x)^{-1} = 5(1 + x + 1x^2 \ldots)$
Add the series for both fractions.
(c) Need $|x| < 1$ and $|2x| < 1$
Answer: (a) $\frac{5}{(1 - x)} + \frac{1}{(1 - 2x)}$ (b) $6 + 7x + 9x^2$ (c) $|x| < \frac{1}{2}$