Day 235 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 128.4 m above the sea at 10 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 6, vertical component 8 (upwards)
(a) $-128.4 = 8t - 4.9t^2$, so $4.9t^2 - 8t - 128.4 = 0$
$t = [8 + \sqrt{2580.64}] \div 9.8 = 6$ (taking the positive root)
(b) Distance = 6×6
(c) Vertical speed = $8 - 9.8\times6 = -50.8$; speed = $\sqrt{(6^2 + 50.8^2)}$
Answer: (a) 6 s (b) 36 m (c) 51.2 m s^{-1}

2 Proving trigonometric identities (All boards)

Prove the identity
$\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot\theta$

$$\text{LHS} = \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta} = \frac{1 + \cos 2\theta}{\sin 2\theta}$$

$$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{RHS}$$

Answer: Proved

3 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg carries a person of mass 80 kg. The cable tension is 3168 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 330 kg), taking upwards as positive: $T - 330g = 330a$
$3168 - 3234 = 330a$, so $a = -0.2$
(a) a is negative, so the acceleration is 0.2 m s^{-2} downwards
(b) Person: $R - 80g = 80a$, so $R = 80(9.8 - 0.2)$
Answer: (a) 0.2 m s^{-2} downwards (b) 768 N

4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{3x-2} = 4^{3x+3}$.

Write both sides as powers of 2.
$(2^3)^{3x-2} = (2^2)^{3x+3}$
$3(3x - 2) = 2(3x + 3)$
$9x - 6 = 6x + 6$
$3x = 12$
Answer: x = 4

5 Equations with sec x and tan x (All boards)

Solve $\sec^2 x - 7\tan x + 11 = 0$
for $0° \le x \le 360°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $\tan^2 x - 7\tan x + 12 = 0$
$\tan x = 3$ or $\tan x = 4$
$\tan x = 3$: $x = 71.6°, 251.6°$
$\tan x = 4$: $x = 76.0°, 256.0°$
Answer: x = 71.6°, 76.0°, 251.6°, 256.0°

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6 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{3}{x-2} - 2$, is translated by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = 2, y = −2

Translate by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$: x = 1, y = −1

New curve: y = $\frac{3}{x-1} - 1$

y = 0: $\frac{3}{x-1} = 1$, so x − 1 = 3

Answer: x = 1, y = −1; crosses x-axis at (4, 0)

7 Smallest number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a customer buys a drink is 0.08 on each independent trial.

Find the least number of trials needed so that the probability of this happening at least once is greater than 0.99.

P(at least one) = $1 - 0.92^n > 0.99$

$0.92^n < 0.01$

$n > \frac{\log 0.01}{\log 0.92} = 55.2$

Least integer n = 56

Answer: 56

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{3} - 1) = 4 + \sqrt{3}$, writing x in the form $p + q\sqrt{3}$.

$x = \frac{4 + \sqrt{3}}{\sqrt{3} - 1}$

Multiply top and bottom by $\sqrt{3} + 1$.

Numerator: $(4 + \sqrt{3})(\sqrt{3} + 1) = 7 + 5\sqrt{3}$

Denominator: 3 − 1 = 2

Answer: $x = \frac{7}{2} + \frac{5}{2}\sqrt{3}$

9 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(21, p). H_0: p = 0.3 is tested against H_1: p ≠ 0.3 at the 5% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. P(X ≤ 1) = 0.00559, P(X ≤ 2) = 0.0271

P(X ≥ 12) = 0.00874, P(X ≥ 11) = 0.0264

Critical region: X ≤ 1 or X ≥ 12

(b) Actual level = 0.00559 + 0.00874 = 0.0143

Answer: (a) X ≤ 1 or X ≥ 12 (b) 0.0143

10 Partial fractions and integration (All boards)

(a) Express $\frac{2x + 9}{(x + 3)(x + 6)}$ in partial fractions.

(b) Hence find $\int_0^4 \frac{2x + 9}{(x + 3)(x + 6)}\,dx$, giving the answer in the form ln k.

$2x + 9 \equiv A(x + 6) + B(x + 3)$: x = −6 gives A = 1; x = −3 gives B = 1

$I = [\ln(x + 3) + \ln(x + 6)]_0^4$

$= \ln \frac{7}{3} + \ln \frac{10}{6}$

$= \ln \frac{35}{9}$

Answer: (a) $\frac{1}{x + 3} + \frac{1}{x + 6}$; (b) $\ln \frac{35}{9}$