Day 234 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(-3 + 3x)^9$.

Term in x^2: ${}^9C_2 \times (-3)^7 \times (3x)^2$
$= 36 \times -2187 \times 9$
Answer: -708588

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 320 items in a list are numbered 1 to 320.
A systematic sample of 16 items is taken, starting with a random item among the first few.
The first item selected is number 4.
Find the sampling interval and the number of the last item selected.

Interval = 320 ÷ 16 = 20
Items selected: 4, 24, 44, ...
Last = 4 + (16 − 1) × 20 = 304
Answer: Interval 20, last item 304

3 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta + 10\tan\theta + 6 = 0$
for $0° \le \theta \le 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta + 10\tan\theta + 8 = 0$
$\tan\theta = -1$ or $\tan\theta = -4$
$\tan\theta = -1$: $\theta = 135.0°$
$\tan\theta = -4$: $\theta = 104.0°$
Answer: θ = 104.0°, 135.0°

4 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 14.4 m above the sea at 20 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 12, vertical component 16 (upwards)
(a) $-14.4 = 16t - 4.9t^2$, so $4.9t^2 - 16t - 14.4 = 0$
$t = [16 + \sqrt{538.24}] \div 9.8 = 4$ (taking the positive root)
(b) Distance = 12×4
(c) Vertical speed = $16 - 9.8\times4 = -23.2$; speed = $\sqrt{(12^2 + 23.2^2)}$
Answer: (a) 4 s (b) 48 m (c) 26.1 m s^{-1}

5 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(21, 0.2).
Find the smallest value of c such that $P(X \ge c) < 0.05$.

$P(X \ge 8) = 1 - P(X \le 7) = 0.0431$
$P(X \ge 7) = 1 - P(X \le 6) = 0.109$
$0.0431 < 0.05$ but $0.109 > 0.05$
Answer: c = 8

6 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 117.6 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.
$4.9t^2 - 4.9(t - 2)^2 = 117.6$
$4.9(4t - 4) = 117.6$, so t = 7
Height = $4.9\times7^2 = 240.1$
Answer: 240.1 m

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7 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 + 2xy + 4y^2 = 33$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{6x + 2y}{2x + 8y}$
Horizontal tangent: $6x + 2y = 0$, so $y = -3x$
Substitute: $33x^2 = 33$, so $x^2 = 1$ and $x = \pm 1$
Points: $(1, -3)$ and $(-1, 3)$
Answer: (1, −3) and (−1, 3)

8 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x + 3y = k$ is a tangent to the circle
$(x + 3)^2 + (y - 1)^2 = 36$.
Find the two possible values of k.

Centre $(-3, 1)$, radius 6
Distance from centre to the line = radius:
$\frac{|-9 - k|}{5} = 6$
$|-9 - k| = 30$
Answer: k = −39 or k = 21

9 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{4} u_n + 3$.
(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{4} L + 3$ gives $L = \frac{3}{1 - \frac{1}{4}} = 4$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-4) \times \left(\frac{1}{4}\right)^{n-1}$

$|-4| \times \left(\frac{1}{4}\right)^{n-1} < \frac{1}{10}$
Check: n = 3 gives 0.25000, n = 4 gives 0.06250
Answer: (a) L = 4 (b) n = 4

10 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3\sec t$, $y = 2\tan t$.
Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{6}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{2}{3\sin t}$

$t = \frac{\pi}{6}$: gradient $m = \frac{4}{3}$, point $(2\sqrt{3}, \frac{2\sqrt{3}}{3})$

$c = y - mx = -2\sqrt{3}$
Answer: $\frac{dy}{dx} = \frac{2}{3\sin t}$; $y = \frac{4}{3}x - 2\sqrt{3}$