Day 233 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2\theta - \sin\theta - 2 = 0$
for $0° \leq \theta \leq 180°$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$: $2\sin^2\theta + \sin\theta = 0$

Solving the quadratic: $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$ has no solutions ($|\sin\theta| \leq 1$)

$\sin\theta = 0$: $\theta = 0°, 180°$

Answer: $\theta = 0°, 180°$

2 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of the word ORDINAL have all the vowels next to each other?

Treat the 3 vowels as one block: 5 items to arrange in 5! ways
The vowels can be ordered within the block in 3! ways
Total = 5! × 3! = 720

Answer: 720

3 Factor theorem: roots with surds (All boards)

$f(x) = x^3 + 2x^2 - 9x - 4$.
(a) Show that $(x + 4)$ is a factor of $f(x)$.
(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(-4) = -64 + 32 + 36 - 4 = 0$, so $(x + 4)$ is a factor.

$f(x) = (x + 4)(x^2 - 2x - 1)$

$x^2 - 2x - 1 = 0$ gives $x = \frac{2 \pm \sqrt{8}}{2}$

$\sqrt{8} = 2\sqrt{2}$

Answer: $x = -4$ or $x = 1 \pm \sqrt{2}$

4 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D \times 6 = 25\,g \times 3.5 + 8\,g \times 8.5$

$R_D = 254$

Resolving vertically: $R_C + R_D = 33\,g = 323.4$

$R_C = 323.4 - 254$

Answer: $R_C = 69.4$ N, $R_D = 254$ N

5 Geometric series: smallest n (All boards)

A geometric sequence has first term 100 and common ratio $\frac{9}{10}$.

Find the first term of the sequence that is less than 1.

$100 \times \left(\frac{9}{10}\right)^{n-1} < 1$

$\left(\frac{9}{10}\right)^{n-1} < \frac{1}{100}$

$(n - 1)\log\frac{9}{10} < \log\frac{1}{100}$; dividing by the negative log reverses the inequality:

$n - 1 > 43.71$, so $n = 45$

Check: $u_{44} = 1.078$, $u_{45} = 0.970$

Answer: n = 45 (the 45th term)

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6 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of

$y = \frac{x^2 + 21}{x + 2}$.

$\frac{dy}{dx} = \frac{(x + 2) \times 2x - (x^2 + 21) \times 1}{(x + 2)^2} = \frac{x^2 + 4x - 21}{(x + 2)^2}$

$\frac{dy}{dx} = 0$: $x^2 + 4x - 21 = 0$, so $(x - 3)(x + 7) = 0$

$x = 3$: $y = 6$; $x = -7$: $y = -14$

Answer: (−7, −14), (3, 6)

7 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 + 4t)\mathbf{i} + (4t^2 - 8t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 3, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 + 4)\mathbf{i} + (8t - 8)\mathbf{j}$

(a) $t = 3$: $\mathbf{v} = 31\mathbf{i} + 16\mathbf{j}$

Speed $= \sqrt{(31^2 + 16^2)} = \sqrt{1217}$

(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $8t - 8 = 0$, so $t = 1$

Answer: (a) 34.9 m s^{-1} (b) t = 1

8 Three-set Venn diagram counts (All boards)

Of 33 students, 18 study Maths (M), 15 Physics (P) and 8 Chemistry (C).
8 study M and P, 3 study M and C, 4 study P and C, and 2 study all three.
One student is chosen at random. Given that a student does not study Maths, find the probability that they study Physics.

Venn regions: M only 9, P only 5, C only 3,
MP only 6, MC only 1, PC only 2, all three 2, outside all 5

Not Maths: $33 - 18 = 15$

Physics but not Maths: $15 - 8 = 7$

Probability $= \frac{7}{15}$

Answer: $\frac{7}{15}$

9 Equations with sec x and tan x (All boards)

Solve $2\sec^2 x + \tan x - 8 = 0$
for $0° \leq x \leq 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $2\tan^2 x + \tan x - 6 = 0$

$\tan x = \frac{3}{2}$ or $\tan x = -2$

$\tan x = \frac{3}{2}$: $x = 56.3°$

$\tan x = -2$: $x = 116.6°$

Answer: x = 56.3°, 116.6°

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-7x + 1}{(1 - x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 + 2x)}$: $A = -2$, $B = 3$

(b) $-2(1 - x)^{-1} = -2(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need $|x| < 1$ and $|2x| < 1$

Answer: (a) $\frac{-2}{(1 - x)} + \frac{3}{(1 + 2x)}$ **(b)** $1 - 8x + 10x^2$ **(c)** $|x| < \frac{1}{2}$