Day 232 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = (3x + 6)^5$.

Find $\frac{dy}{dx}$ and the gradient of the curve at $x = 3$.

Let $u = 3x + 6$, so $y = u^5$, $\frac{du}{dx} = 3$, $\frac{dy}{du} = 5u^4$

$\frac{dy}{dx} = 15(3x + 6)^4$

At $x = 3$: $u = 15$, so gradient = 759375

Answer: $\frac{dy}{dx} = 15(3x + 6)^4$**; gradient = 759375**

2 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (7\mathbf{i} - 6\mathbf{j})$ N, $F_2 = (-12\mathbf{i} + 9\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 3 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

i: $7 + (-12) + a = 0$, so $a = 5$

j: $-6 + 9 + b = 0$, so $b = -3$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-12^2 + 9^2)} = 15$ N

$a = 15 \div 3$

Answer: (a) a = 5, b = −3 (b) 5 m s^{-2}

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^3 \frac{3}{4x + 3}\,dx$.

Integrate: $[\frac{3}{4}\ln(4x + 3)]$

Substitute the limits and subtract: $\frac{3 \ln 5}{4}$

Answer: $\frac{3 \ln 5}{4}$

4 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 59 and standard deviation 5.
Find (a) P(X > 54.5) (b) P(54.5 < X < 57), each to 3 s.f.

(a) $z = \frac{54.5 - 59}{5} = -0.900$, $P(Z > -0.900) = 0.816$

(b) $z_1 = -0.900$, $z_2 = \frac{57 - 59}{5} = -0.400$

$P(54.5 < X < 57) = \Phi(-0.400) - \Phi(-0.900) = 0.161$

Answer: (a) 0.816 (b) 0.161

5 Logarithmic equations (All boards)

Solve $\log_3(x + 1) + \log_3(x - 5) = 3$.

Combine: $\log_3[(x + 1)(x - 5)] = 3$, so $(x + 1)(x - 5) = 3^3 = 27$

$x^2 - 4x - 32 = 0$: $(x - 8)(x + 4) = 0$

$x = 8$ or $x = -4$; $x = -4$ makes $x - 5$ negative, so it is rejected

Answer: x = 8

6 Binomial expansion: rational powers (All boards)

Expand $(1 - 2x)^{-3}$ in ascending powers of x up to and including the term in x^3.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -3$, $y = -2x$

Coefficients of 1, x, x^2, x^3: 1, 6, 24, 80

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + 6x + 24x^2 + 80x^3$; $|x| < \frac{1}{2}$

7 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 + 6t)\mathbf{i} + (2t^2 - 4t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 4, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 + 6)\mathbf{i} + (4t - 4)\mathbf{j}$
(a) t = 4: $\mathbf{v} = 54\mathbf{i} + 12\mathbf{j}$
Speed = $\sqrt{(54^2 + 12^2)} = \sqrt{3060}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $4t - 4 = 0$, so t = 1
Answer: (a) 55.3 m s^{-1} (b) t = 1

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(4 + x)^{\frac{1}{2}}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(4 + x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1 + \frac{1x}{4})^{\frac{1}{2}}$

$4^{\frac{1}{2}} = 2$

Expand $(1 + y)^{\frac{1}{2}}$ with $y = \frac{1x}{4}$

Valid for $|\frac{1x}{4}| < 1$, i.e. $|x| < 4$

Answer: $2 + \frac{1}{4}x - \frac{1}{64}x^2$; $|x| < 4$

9 Recurrence relations: limits (All boards)

$u_1 = 18$ and $u_{n+1} = -\frac{1}{2}u_n + 12$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = -\frac{1}{2}L + 12$ gives $L = \frac{12}{1 - (-\frac{1}{2})} = 8$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = 10 \times (-\frac{1}{2})^{n-1}$

$|10| \times \frac{1}{2}^{n-1} < \frac{1}{100}$
Check: n = 10 gives 0.01953, n = 11 gives 0.00977
Answer: (a) L = 8 (b) n = 11

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10 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The marks of two groups: class A has n = 28, mean 47, sd 7;
class B has n = 35, mean 20, sd 4.
Find the mean and standard deviation of all 63 values (3 s.f. for the sd).

$\Sigma x = 28 \times 47 + 35 \times 20 = 2016$, mean = 32

Σx^2 (A) $= 28(7^2 + 47^2) = 63224$

Σx^2 (B) $= 35(4^2 + 20^2) = 14560$

Variance $= \frac{77784}{63} - 32^2 = 210.666667$, so sd = 14.5

Answer: mean 32, sd 14.5