Day 231 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (5\mathbf{i} + 9\mathbf{j})$ N, $F_2 = (-8\mathbf{i} + 15\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 4 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

$\mathbf{i}$: $5 + (-8) + a = 0$, so $a = 3$

$\mathbf{j}$: $9 + 15 + b = 0$, so $b = -24$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-8^2 + 15^2)} = 17$ N

$a = 17 \div 4$

Answer: (a) $a = 3$, $b = -24$ (b) 4.25 m s^{-2}

2 Arithmetic series: finding n (All boards)

An arithmetic series has first term 9 and common difference 4.
The sum of the first n terms is 429. Find n.

$\frac{n}{2}(2 \times 9 + (n - 1) \times 4) = 429$

$4n^2 + 14n = 858$

$4n^2 + 14n - 858 = 0$

$(n - 13)(4n + 66) = 0$, so $n = 13$ or $n = -\frac{33}{2}$

n must be a positive integer, so $n = 13$

Answer: 13

3 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has lower quartile 27 and upper quartile 32.
A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.
Which of 15, 22, 29, 41 are outliers?

IQR = 32 − 27 = 5, so 1.5 × IQR = 7.5

Lower limit = 27 − 7.5 = 19.5

Upper limit = 32 + 7.5 = 39.5

Outliers are values below 19.5 or above 39.5

Answer: 15, 41

4 Change of sign to locate a root (All boards)

$f(x) = x^3 - 2x - 10$.
Show that the equation $f(x) = 0$ has a root α in the interval [2, 3].

f(x) is continuous on the interval

$f(2) = -6.00 < 0$

$f(3) = 11.0 > 0$

A change of sign, so a root lies in [2, 3]

Answer: Sign change: $f(2) = -6.00$, $f(3) = 11.0$

5 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 8 and variance 6.
(a) Find n and p.
(b) Find P(X = 9) to 3 significant figures.

(a) $np = 8$ and $npq = 6$

$q = \frac{npq}{np} = \frac{6}{8} = 0.75$, so $p = 0.25$

$n = \frac{8}{0.25} = 32$

(b) $P(X = 9) = C(32, 9)(0.25)^9(0.75)^{23} = 0.143$

Answer: (a) $n = 32$, $p = 0.25$ (b) 0.143

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6 Double and triple angle values (All boards)

θ is acute and $\tan\theta = \frac{7}{3}$.

Show that $\cos 2\theta = -\frac{20}{29}$ and find the exact value of $\sin 2\theta$.

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{7}{3}^2}{1 + \frac{7}{3}^2} = -\frac{20}{29}$$

$$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2 \times \frac{7}{3}}{1 + \frac{7}{3}^2} = \frac{21}{29}$$

Answer: $\cos 2\theta = -\frac{20}{29}$, $\sin 2\theta = \frac{21}{29}$

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{14} in the expansion of

$(3x^3 + \frac{3}{x})^6$.

General term: ${}^6C_r\,(3x^3)^{6-r}\,(3x^{-1})^r$

Power of x: $3(6 - r) - 1r = 14$

$18 - 4r = 14$ so $r = 1$

Coefficient = ${}^6C_1 \times 3^5 \times 3^1 = 4374$

Answer: 4374

8 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-x^2 + 3xy + 3y^2 - 4x + 2y = 5$.
Show that P$(-1, 1)$ lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(-2x + 3y - 4) + (3x + 6y + 2)\frac{dy}{dx} = 0$

At P$(-1, 1)$: $1 + (5)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{1}{5}$

Tangent: $y - 1 = (-\frac{1}{5})(x + 1)$, i.e. $x + 5y - 4 = 0$

Answer: $x + 5y - 4 = 0$

9 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(-\mathbf{i})$ m and $(4\mathbf{i})$ m and velocities $(5\mathbf{i} + 2\mathbf{j})$ m s^{-1} and $(3\mathbf{i} + 3\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$\mathrm{PQ} = \mathrm{r_P} - \mathrm{r_Q} = (-5\mathbf{i}) + t(2\mathbf{i} - \mathbf{j})$

$d^2 = (2t - 5)^2 + (-t)^2 = 5t^2 - 20t + 25$

(a) $d(d^2)/dt = 0$: $10t + -20 = 0$, so $t = 2$

(b) At t = 2: $\mathrm{PQ} = -\mathbf{i} - 2\mathbf{j}$, so $d = \sqrt{5}$

Answer: (a) t = 2 s (b) 2.24 m

10 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A$(-1, 0)$ and passes through B$(2, 3)$.
Find the equation of the circle.

Touches the x-axis at A, so the centre is $(-1, r)$ and the radius is r.

B on the circle: $(3)^2 + (3 - r)^2 = r^2$

$9 + 9 - 6r = 0$ so $r = 3$

Answer: $(x + 1)^2 + (y - 3)^2 = 9$