Day 230 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3(t - \sin t)$, $y = 3(1 - \cos t)$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{3}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{\sin t}{1 - \cos t}$

$t = \frac{\pi}{3}$: gradient $m = \sqrt{3}$, point $(\frac{2\pi - 3\sqrt{3}}{2}, \frac{3}{2})$

$c = y - mx = 6 - \sqrt{3}\pi$

Answer: $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$; $y = \sqrt{3}x + 6 - \sqrt{3}\pi$

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(4 + 2x)^7$.

Term in x^4: ${}^7C_4 \times 4^3 \times (2x)^4$

$= 35 \times 64 \times 16$

Answer: 35840

3 Differentiating sec, cosec, tan and cot (All boards)

$y = \cot(3x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{18}$.

$\frac{dy}{dx} = -3\text{cosec}^2(3x)$

At $x = \frac{\pi}{18}$ the angle $3x = \frac{\pi}{6}$, so the gradient is -12

Answer: $\frac{dy}{dx} = -3\text{cosec}^2(3x)$; -12

4 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the square of any integer is of the form $5k$, $5k + 1$ or $5k - 1$.

Any integer is $5m$, $5m \pm 1$ or $5m \pm 2$.

$(5m)^2 = 5(5m^2)$

$(5m \pm 1)^2 = 25m^2 \pm 10m + 1 = 5(5m^2 \pm 2m) + 1$

$(5m \pm 2)^2 = 25m^2 \pm 20m + 4 = 5(5m^2 \pm 4m + 1) - 1$

All cases covered.

Answer: squares are of the form $5k$, $5k + 1$ or $5k - 1$

5 Equations with sec x and tan x (All boards)

Solve $3\sec^2 x + 9\tan x + 3 = 0$

for $-180° \leq x \leq 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $3\tan^2 x + 9\tan x + 6 = 0$

$\tan x = -2$ or $\tan x = -1$

$\tan x = -2$: $x = -63.4°, 116.6°$

$\tan x = -1$: $x = -45.0°, 135.0°$

Answer: $x = -63.4°, -45.0°, 116.6°, 135.0°$

6 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-1-4\sqrt{6}}{4+2\sqrt{6}}$ in the form $p+q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by $4-2\sqrt{6}$.
Denominator: $(4+2\sqrt{6})(4-2\sqrt{6}) = -8$
Numerator: $(-1-4\sqrt{6})(4-2\sqrt{6}) = 44-14\sqrt{6}$
So the expression is $\frac{44-14\sqrt{6}}{-8}$.

Answer: $-\frac{11}{2}+\frac{7}{4}\sqrt{6}$

7 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 50 values of x, the coded variable $y = \frac{x-40}{2}$ gives

$\Sigma y = 175$ and $\Sigma y^2 = 925$.
Find the mean and standard deviation of x.

$\bar{y} = \frac{175}{50} = 3.5$

$\sigma_y^2 = \frac{925}{50} - 3.5^2 = 6.25$, so $\sigma_y = 2.5$

$x = 2y + 40$, so mean $= 2 \times 3.5 + 40 = 47$
Standard deviation $= 2 \times 2.5 = 5$
Answer: mean 47, sd 5

8 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$; friction $F = \mu R = 0.1 \times 3g\cos\alpha$
B: $9g - T = 9a$
A: $T - 3g\sin\alpha - F = 3a$
Adding: $g(9 - \frac{15}{13} - \frac{18}{65}) = 12a$
(a) $a = 6.18$; (b) $T = 9(g - a)$
Answer: (a) 6.18 m s^{-2} (b) 32.6 N

9 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 7.7 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{4}{3}$. The coefficient of friction is 0.5. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration $= g(\sin\alpha + \mu\cos\alpha) = 9.8(0.8 + 0.5 \times 0.6) = 10.78$
(a) $0 = 7.7^2 - 2 \times 10.78 \times d$, so $d = 2.75$
Down: $\tan\alpha = \frac{4}{3} > \mu$, so it slides back; acceleration $= g(\sin\alpha - \mu\cos\alpha) = 4.9$
(b) $v^2 = 2 \times 4.9 \times 2.75 = 26.95$
$v = \sqrt{26.95}$
Answer: (a) 2.75 m (b) 5.19 m s^{-1}

10 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily mean wind speed is modelled by N(7.4, 2.9^2).
(a) Find the probability that on a randomly chosen day the value exceeds 12 knots.
(b) Estimate the number of such days in a period of 62 days.

(a) $z = \frac{12 - 7.4}{2.9} = 1.59$

$P(Z > 1.59) = 0.0563$

(b) $62 \times 0.0563 = 3.49 \approx 3$

Answer: (a) 0.0563 (b) 3 days