Day 23 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is N(171, 7^2).
Find the symmetric limits about the mean between which the central 95% of values lie (3 s.f.).

Each tail has area 0.025, so P(Z < z) = 0.975
z = 1.96
Limits: 171 ± 1.96 × 7
= 157 and 185
Answer: 157 to 185

2 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (4 kg) hang on a light string over a smooth pulley, A being 1.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: 6 g − T = 6 a; B: T − 4 g = 4 a
a = 2 g ÷ 10 = 1.96, so T = 4(g + a) = 47.04
When A lands, B has risen 1.5 m with $v^2 = 2\times1.96\times1.5 = 5.88$
B then moves freely under gravity: extra height = $v^2 \div (2\times9.8) = 0.3$
Total = 1.5 + 0.3
Answer: (a) 47.04 N (b) 1.8 m

3 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (5, 5).
Find the coordinates of the corresponding point on:
(a) y = f(x) + 5
(b) y = f(−x)
(c) y = f(x + 1)

(a) y = f(x) + 5: (5, 10)
(b) y = f(−x): (−5, 5)
(c) y = f(x + 1): (4, 5)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (5, 10); (b) (−5, 5); (c) (4, 5)

4 Product rule: stationary points (All boards)

$y = (4x + 6)e^{-2x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4\,e^{-2x} - 2(4x + 6)e^{-2x} = (-8x - 8)e^{-2x}$

$e^{-2x} > 0$, so $\frac{dy}{dx} = 0$ gives $-8x - 8 = 0$ and $x = -1$

$y = 2e^2$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(-1, 2e^2)$, maximum

5 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function f(x) = kx(3 − x) for 0 ≤ x ≤ 3, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find P(X > 1).

(a) $\int_0^3 f(x)\,dx = 1$ gives $\frac{9}{2}k = 1$, so $k = \frac{2}{9}$

(b) $E(X) = \int x\,f(x)\,dx = \frac{3}{2}$

(c) $P(X > 1) = \int_1^3 f(x)\,dx = \frac{20}{27}$

Answer: (a) $k = \frac{2}{9}$ (b) $\frac{3}{2}$ (c) $\frac{20}{27}$

6 Quotient rule: stationary points (All boards)

Find the exact coordinates of the stationary point of

$y = \frac{e^{2x}}{x}$ for $x > 0$.

$\frac{dy}{dx} = \frac{x \times 2e^{2x} - e^{2x} \times 1}{x^2} = \frac{e^{2x}(2x - 1)}{x^2}$

$e^{2x} > 0$, so $\frac{dy}{dx} = 0$ when $2x = 1$, giving $x = \frac{1}{2}$

$y = e^1 \div \frac{1}{2} = 2e$

Answer: $(\frac{1}{2}, 2e)$

7 Maximum speed from velocity function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (-3t^2 + 24t)$ m s^{-1} at time t s, $t \geq 0$, starting at O. Find (a) its greatest speed, (b) its distance from O when t = 8, (c) the time at which it returns to O.

(a) $dv/dt = 24 - 6t = 0$, so $t = 4$ (a maximum since the t^2 coefficient is negative)
$v = 24 \times 4 - 3 \times 4^2 = 48$
(b) $x = \int v \, dt = 12\,t^2 - 1\,t^3$
$x(8) = 256$
(c) $x = 0$ when $t^2(12 - 1\,t) = 0$, so $t = 12$

Answer: (a) 48 m s^{-1} (b) 256 m (c) 12 s

8 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 10 and the coefficient of x^2 is 45. Find n and k.

$nk = 10$ and $\frac{n(n-1)}{2}k^2 = 45$

$(n - 1)k = \frac{2 \times 45}{10} = 9$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = 10 - 9 = 1$

$n = \frac{10}{1} = 10$

Answer: n = 10, k = 1

9 Modulus inequalities (All boards)

Solve $|x + 7| > |4x + 8|$.

Square both sides: $(x + 7)^2 > (4x + 8)^2$
$((x + 7) - (4x + 8))((x + 7) + (4x + 8)) > 0$
$(-3x - 1)(5x + 15) > 0$

Critical values $x = -3$ and $x = -\frac{1}{3}$; sketch the (positive or negative) quadratic.

Answer: $-3 < x < -\frac{1}{3}$

10 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{5}{13}$ and $\cos B = -\frac{7}{25}$.

Find the exact values of tan(A+B) and cos(A+B).

$\cos A = \frac{12}{13}$ (A acute), $\sin B = \frac{24}{25}$ (B obtuse), so $\tan A = \frac{5}{12}$, $\tan B = -\frac{24}{7}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{5}{12} + (-\frac{24}{7})}{1 - \frac{5}{12} \times (-\frac{24}{7})} = -\frac{253}{204}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{12}{13} \times (-\frac{7}{25}) - \frac{5}{13} \times \frac{24}{25} = -\frac{204}{325}$$

Answer: $\tan(A+B) = -\frac{253}{204}$, $\cos(A+B) = -\frac{204}{325}$