Day 229 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $5^{3x-3} = 7^{2x-3}$.
Give your answer to 3 significant figures.

Take logs: $(3x - 3)\ln 5 = (2x - 3)\ln 7$
Collect x terms: $x(3\ln 5 - 2\ln 7) = -3\ln 7 + 3\ln 5$
$x = \dfrac{-3\ln 7 + 3\ln 5}{3\ln 5 - 2\ln 7}$
$x = -1.08$
Answer: $x = -1.08$

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 2 m s^{-1} and accelerates uniformly. She travels 200 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $200 = 2\times20 + \frac{1}{2}\times a\times20^2$, so $a = 0.8$
(b) $v = u + at = 2 + 0.8\times20 = 18$
(c) For the midpoint, $s = 100$: $v^2 = u^2 + 2as = 2^2 + 2\times0.8\times100$
$v^2 = 164$
Answer: (a) 0.8 m s^{-2} (b) 18 m s^{-1} (c) 12.8 m s^{-1}

3 Arc length and sector area (All boards)

A sector of a circle has arc length 16.5 cm and area 90.75 cm^2.
Find the radius of the circle and the angle of the sector, in radians.

Area $= \frac{1}{2}rs$: $90.75 = \frac{1}{2} \times r \times 16.5$
$r = \dfrac{2 \times 90.75}{16.5} = 11$ cm
$\theta = \dfrac{s}{r} = \dfrac{16.5}{11} = 1.5$ rad
Answer: r = 11 cm, θ = 1.5 rad

4 Inverse of a rational function (All boards)

$f(x) = \dfrac{3x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.
Find $f^{-1}(x)$ and state its domain.

$y = \dfrac{3x+3}{2x-1}$: $y(2x - 1) = 3x + 3$
$2xy - y = 3x + 3$
$x(2y - 3) = y + 3$
$x = \dfrac{y+3}{2y-3}$, then swap x and y
Answer: $f^{-1}(x) = \dfrac{x+3}{2x-3}$, $x \neq \dfrac{3}{2}$

5 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 49 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.
$4.9t^2 - 4.9(t - 2)^2 = 49$
$4.9(4t - 4) = 49$, so $t = 3$
Height $= 4.9\times3^2 = 60.025$
Answer: 60.025 m

6 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is N(68, 4^2). An egg is classed as 'special' if its mass is less than 64.
9 eggs are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X < 64) = P(Z < −1.00) = 0.159
Y ~ B(9, 0.159)
P(Y = 3) = C(9, 3)$(0.159)^3(0.841)^6$ = 0.119
Answer: 0.119

7 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{2}{x+3}$ + 2, is translated by $\begin{pmatrix}-1\\1\end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = −3, y = 2

Translate by $\begin{pmatrix}-1\\1\end{pmatrix}$: x = −4, y = 3

New curve: y = $\frac{2}{x+4}$ + 3

y = 0: $\frac{2}{x+4}$ = −3, so x + 4 = $-\frac{2}{3}$

Answer: x = −4, y = 3; crosses x-axis at $(-\frac{14}{3}, 0)$

8 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a patient recovers is 0.2. In a sample of 18, the number of successes is 6.
Test at the 5% level whether p has increased.

H_0: p = 0.2, H_1: p > 0.2; X ~ B(18, 0.2)
P(X ≥ 6) = 1 − P(X ≤ 5) = 0.133
0.133 > 0.05
Do not reject H_0: insufficient evidence of a change in p.
Answer: p-value 0.133; do not reject H_0

9 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of

y = $\frac{x^2}{x-1}$.

$\frac{dy}{dx} = \frac{(x-1)\times 2x - x^2\times 1}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$

$\frac{dy}{dx}$ = 0 when x = 0 or x = 2

x = 0: y = 0; x = 2: y = $\frac{4}{1}$ = 4

Answer: (0, 0), (2, 4)

10 Partial fractions and binomial expansion (All boards)

f(x) = $\frac{12x+3}{(1-x)(1+2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) f(x) = $\frac{A}{(1-x)} + \frac{B}{(1+2x)}$: A = 5, B = −2

(b) $5(1-x)^{-1} = 5(1 + x + 1x^2 ...)$
Add the series for both fractions.

(c) Need |x| < 1 and |2x| < 1

Answer: (a) $\frac{5}{(1-x)} + \frac{-2}{(1+2x)}$ (b) $3 + 9x - 3x^2$ (c) $|x| < \frac{1}{2}$