Day 228 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a maximum point at $(-4, 3)$.
Find the coordinates of the corresponding point on:
(a) $y = f(4x)$
(b) $y = f(x + 2)$
(c) $y = f(-x)$

(a) $y = f(4x)$: $(-1, 3)$
(b) $y = f(x + 2)$: $(-6, 3)$
(c) $y = f(-x)$: $(4, 3)$
A stretch or translation moves the point in the stated direction only.
Answer: (a) $(-1, 3)$; (b) $(-6, 3)$; (c) $(4, 3)$

2 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$, and passes through A, 18 m horizontally from O and 19.1 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $18 = U\cos\alpha \times t$, so $t = 18 \div (0.6\,U)$
Vertical: $19.1 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$19.1 = 24 - 4.9\times 18^2 \div (0.36\,U^2)$
$U^2 = 4410 \div 4.9 = 900$, $U = 30$
Answer: 30 m s^{-1}

3 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4^{3x-2} = 32^{x+2}$.

Write both sides as powers of 2.
$(2^2)^{3x-2} = (2^5)^{x+2}$
$2(3x - 2) = 5(x + 2)$
$6x - 4 = 5x + 10$
$x = 14$
Answer: $x = 14$

4 Smallest whole number solving an exponential inequality (All boards)

Find the least integer n for which $7^n > 10^6$.

$n \lg 7 > 6$, so $n > \frac{6}{\lg 7} = 7.10$
Since n must be an integer, $n = 8$
Answer: $n = 8$

5 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-4x + 3y = k$ is a tangent to the circle
$(x - 2)^2 + (y + 2)^2 = 36$.
Find the two possible values of k.

Centre $(2, -2)$, radius 6
Distance from centre to the line = radius:
$\frac{|-14 - k|}{5} = 6$
$|-14 - k| = 30$
Answer: $k = -44$ or $k = 16$

6 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}-2\\-2\\-1\end{pmatrix} + t\begin{pmatrix}3\\0\\0\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, −5, −3) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix}3t-3\\3\\2\end{pmatrix}$

PF·d = 0 gives 9t − 9 = 0, so t = 1

F = (1, −2, −1); $\mathrm{PF} = \begin{pmatrix}0\\3\\2\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{13}$

Answer: F(1, −2, −1), distance $\sqrt{13}$

7 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 31.36 m from the start, after 4 s and again after 8 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration a = g sin α (unknown). $s = ut - \frac{1}{2}at^2$ at both times:

$u\times4 - \frac{1}{2}a\times4^2 = u\times8 - \frac{1}{2}a\times8^2 = 31.36$

Subtracting: $u(8-4) = \frac{1}{2}a(64-16)$, so $u = \frac{1}{2}a(12)$

Substituting: $\frac{1}{2}a(12)(4) - \frac{1}{2}a\times4^2 = 31.36$, so $\frac{1}{2}a\times32 = 31.36$, a = 1.96

(a) u = 0.98×12 = 11.76

(b) 9.8 sin α = 1.96, sin α = 0.2, α = 11.5°

Answer: (a) 11.76 m s^{-1} (b) 11.5°

8 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ N(51, 4^2). Find P(X > 60 | X > 55) to 3 significant figures.

Since X > 60 implies X > 55: $\mathrm{P} = \frac{\mathrm{P}(X>60)}{\mathrm{P}(X>55)}$

P(X > 60) = P(Z > 2.25) = 0.0122

P(X > 55) = P(Z > 1.00) = 0.159

$\mathrm{P} = \frac{0.0122}{0.159} = 0.0771$

Answer: 0.0771

9 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 7 men and 4 women. A team of 4 is chosen at random.
Find the probability that the team has exactly 2 of the women.

Total teams = C(11, 4) = 330

Favourable = C(4, 2) × C(7, 2) = 6 × 21 = 126

$\mathrm{P} = \frac{126}{330}$

$= \frac{21}{55}$

Answer: $\frac{21}{55}$

10 Tangent through the origin (All boards)

The tangent to the curve y = ln(x) at the point P passes through the origin.
Find the exact coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = \frac{1}{x}$; tangent at x = a: $y - \ln(a) = \frac{1}{a}(x-a)$

Through (0, 0): −ln(a) = −1, so a = e and a = e

P = (e, 1), gradient $\frac{1}{a} = e^{-1}$, so the tangent is $y = e^{-1}x$

Answer: P(e, 1); $y = e^{-1}x$