Day 227 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Logarithmic equations (All boards)

Solve $\log_2(x + 1) + \log_2(x - 1) = 3$.

Combine: $\log_2[(x + 1)(x - 1)] = 3$, so $(x + 1)(x - 1) = 2^3 = 8$

$x^2 - 9 = 0$: $(x - 3)(x + 3) = 0$

$x = 3$ or $x = -3$; $x = -3$ makes $x - 1$ negative, so it is rejected

Answer: $x = 3$

2 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1950 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = $\frac{1}{2}(90 + 40) \times V$

$1950 = 65\,V$, so $V = 30$

(b) Deceleration = $V \div 20 = 30 \div 20$

Answer: (a) $V = 30$ (b) $1.5\ \text{m s}^{-2}$

3 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 10 pairs of (x, y): $\Sigma x = 186$, $\Sigma y = 446$,

$\Sigma x^2 = 4144$, $\Sigma y^2 = 23530$, $\Sigma xy = 9801$.

Calculate the product moment correlation coefficient, and describe the correlation.

$$S_{xx} = 4144 - \frac{186^2}{10} = 684$$

$$S_{yy} = 23530 - \frac{446^2}{10} = 3640$$

$$S_{xy} = 9801 - \frac{186 \times 446}{10} = 1510$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.954$$

This is strong positive correlation

Answer: $r = 0.954$, positive correlation

4 Product rule: stationary points (All boards)

$y = (3x + 1)e^{-3x}$.

Find the exact coordinates of the stationary point and determine its nature.

$$\frac{dy}{dx} = 3\,e^{-3x} - 3(3x + 1)e^{-3x} = (-9x)e^{-3x}$$

$e^{-3x} > 0$, so $\frac{dy}{dx} = 0$ gives $-9x = 0$ and $x = 0$

$y = 1$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: (0, 1), maximum

5 Geometric sequences: finding a and r (All boards)

The 2nd term of a geometric sequence is 6 and the 5th term is 162.

Find the common ratio and the first term.

Find the sum of the first 8 terms.

$$r^3 = \frac{162}{6} = 27 \text{ so } r = 3$$

$a\,r^1 = 6$ so $a = 2$

$$S_8 = \frac{a(r^8 - 1)}{(r - 1)} = \frac{2(6561 - 1)}{2}$$

Answer: $r = 3$, $a = 2$, $S_8 = 6560$

6 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 37 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{24}{7}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity (10.36, 35.52). At time T the velocity is $(10.36, 35.52 - 9.8T)$.
Perpendicular: scalar product = 0: $(10.36)^2 + 35.52(35.52 - 9.8T) = 0$
$37^2 - 9.8T \times 35.52 = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 37 \div (9.8 \times 0.96) = \frac{4625}{1176}$

(b) $h = 35.52 \times \frac{4625}{1176} - 4.9 \times \left(\frac{4625}{1176}\right)^2 = \frac{3607315}{56448}$

Answer: (a) T = 3.93 s (b) 63.9 m

7 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}10\\0\\-3\end{pmatrix} + \lambda\begin{pmatrix}3\\-2\\-3\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}10\\2\\-2\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\3\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $10 + 3\lambda = 10 - 3\mu$; $0 - 2\lambda = 2 + 1\mu$; $-3 - 3\lambda = -2 + 3\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = -36$ meets the circle $(x + 4)^2 + (y + 1)^2 = 9$ at the points A and B. Find the exact length of AB.

Centre $(-4, -1)$, radius 3. Distance from centre to line:

$d = \frac{|-17|}{13} = \frac{17}{13}$

Half-chord$^2 = 9 - \left(\frac{17}{13}\right)^2 = \frac{1232}{169}$

$AB = 2\sqrt{\frac{1232}{169}}$

Answer: $\mathbf{\frac{104\sqrt{77}}{169}}$

9 Poisson over different intervals (CIE)

Faults are reported at random at an average of 10 per hour.
Find the probability that in a 90-minute period there are (a) zero events (b) at least 3.
(c) Find the probability that at least one occurs in each of two separate 90-minute periods.

$\lambda = 10 \times \frac{90}{60} = 15$, $X \sim Po(15)$

(a) $P(X = 0) = e^{-15} = 0.000000306$
(b) $P(X \geq 3) = 1 - P(X \leq 2) = 1.00$
(c) $(1 - 0.000000306)^2 = 1.00$
Answer: (a) 0.000000306 (b) 1.00 (c) 1.00

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10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.01 to estimate $\sqrt{101}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, y = 1x

(a) $1 + \frac{1}{2}x - \frac{1}{8}x^2$

$x = \frac{1}{100}$: $101 = 10^2(1 + \frac{1}{100})$ so $\sqrt{101} = 10 \times (1 + \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{200} - \frac{1}{80000}) = 10.049875$

Answer: (a) $1 + \frac{1}{2}x - \frac{1}{8}x^2$ (b) 10.0499