Day 226 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |x + 6|$ and $y = -\frac{1}{2}x + 6$.

The graphs meet at A and B.
Find the coordinates of A and B.

$|x + 6| = -\frac{1}{2}x + 6$

Solve $x + 6 = \pm(-\frac{1}{2}x + 6)$.

$x = -24$ and $x = 0$ (both checked in the original equation)
$y = 18$ and $y = 6$
Answer: A$(-24, 18)$, B$(0, 6)$

2 Arc length and sector area (All boards)

A sector of a circle of radius 12 cm has angle 30°.
Find, in terms of π, the arc length, the area and the perimeter of the sector.

$\theta = 30° = \frac{\pi}{6}$ radians

Arc length $= r\theta = 12 \times \frac{\pi}{6} = 2\pi$ cm

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times \frac{\pi}{6} = 12\pi$ cm^2

Perimeter = 2r + arc $= 2\pi + 24$ cm

Answer: arc 2π cm, area 12π cm^2, perimeter $2\pi + 24$ cm

3 Geometric series: smallest n (All boards)

A geometric sequence has first term 200 and common ratio $\frac{1}{2}$.

Find the first term of the sequence that is less than 5.

$200 \times \frac{1}{2}^{n-1} < 5$

$\frac{1}{2}^{n-1} < \frac{1}{40}$

$(n - 1) \log \frac{1}{2} < \log \frac{1}{40}$; dividing by the negative log reverses the inequality:

$n - 1 > 5.32$, so $n = 7$
Check: $u_6 = 6.250$, $u_7 = 3.125$
Answer: n = 7 (the 7th term)

4 Poisson probabilities (CIE)

Calls reach a switchboard at a mean rate of 3.5 per day, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 2 (b) at least 2 (c) fewer than 2 (3 s.f.).

$X \sim Po(3.5)$
(a) $P(X = 2) = e^{-3.5}\, 3.5^2/2! = 0.185$
(b) $1 - P(X \leq 1) = 1 - 0.136 = 0.864$
(c) $P(X \leq 1) = 0.136$
Answer: (a) 0.185 (b) 0.864 (c) 0.136

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5 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(2r+3) = 165$, find the value of n.

$\sum_{r=1}^{n}(2r+3) = 2 \times \frac{n(n+1)}{2} + 3n$

$n^2 + 4n = 165$

$2n^2 + 8n - 330 = 0$

Solve and reject the negative root: n = 11

Answer: 11

6 Equations with sec x and tan x (All boards)

Solve $4\sec^2\theta - 10\tan\theta - 28 = 0$
for $-180° \leq \theta \leq 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $4\tan^2\theta - 10\tan\theta - 24 = 0$

$\tan\theta = -\frac{3}{2}$ or $\tan\theta = 4$

$\tan\theta = -\frac{3}{2}$: $\theta = -56.3°, 123.7°$

$\tan\theta = 4$: $\theta = -104.0°, 76.0°$

Answer: $\theta = -104.0°, -56.3°, 76.0°, 123.7°$

7 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-7\mathbf{i} - \mathbf{j})$ N, $F_2 = (15\mathbf{i} - 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 5 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

i: $-7 + 15 + a = 0$, so $a = -8$

j: $-1 + (-8) + b = 0$, so $b = 9$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(15^2 + -8^2)} = 17$ N

$a = 17 \div 5$

Answer: (a) $a = -8$, $b = 9$ (b) 3.4 m s^{-2}

8 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 77 and standard deviation 9.
A value is treated as an outlier if it is more than 3 standard deviations from the mean.
Which of 53, 85, 102, 113 are outliers?

3 standard deviations = 3 × 9 = 27

Lower limit = 77 − 27 = 50

Upper limit = 77 + 27 = 104

Outliers are values below 50 or above 104

Answer: 113

9 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2\theta + 10\cot\theta + 6 = 0$
for $0° < \theta < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $2\cot^2\theta + 10\cot\theta + 8 = 0$

$\cot\theta = -1$ or $\cot\theta = -4$

$\cot\theta = -1$ means $\tan\theta = -1$: $\theta = 135.0°$

$\cot\theta = -4$ means $\tan\theta = -\frac{1}{4}$: $\theta = 166.0°$

Answer: $\theta = 135.0°, 166.0°$

10 Ships on collision courses (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

At 12:00 ship P has position $(12\mathbf{i} + 24\mathbf{j})$ km and velocity $(-\mathbf{i} - 4\mathbf{j})$ km h^{-1}; ship Q has position $(2\mathbf{i} - 6\mathbf{j})$ km and velocity $(\mathbf{i} + 2\mathbf{j})$ km h^{-1}. Show that the ships collide and find the time and position vector of the collision.

At time t hours: $\mathbf{r}_P = (12\mathbf{i} + 24\mathbf{j}) + t(-\mathbf{i} - 4\mathbf{j})$, $\mathbf{r}_Q = (2\mathbf{i} - 6\mathbf{j}) + t(\mathbf{i} + 2\mathbf{j})$

Equal when: $\mathbf{i}$: $12 + -1\,t = 2 + 1\,t$, so $t = 5$

$\mathbf{j}$: $24 + 5\times(-4) = 4$

Check with Q: $\mathbf{i}$: $2 + 5\times1 = 7$, $\mathbf{j}$: $-6 + 5\times2 = 4$

Both components agree at $t = 5$, so they collide at $(7\mathbf{i} + 4\mathbf{j})$ km

Answer: $t = 5$ hours (at 17:00); $\mathbf{r} = (7\mathbf{i} + 4\mathbf{j})$ km