Day 225 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 240 items.
The 4th item chosen is number 22 and the 5th item chosen is number 28.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(28 - 22) \div (5 - 4) = 6$
First item = $22 - 3 \times 6 = 4$
Sample size = $240 \div 6 = 40$
Answer: Interval 6, first 4, size 40

2 Completing the square (All boards)

$f(x) = 3x^2 + 12x + 10$.
(a) Write $f(x)$ in the form $a(x + p)^2 + q$.
(b) State the minimum value of $f(x)$ and the value of x at which it occurs.
(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = 3(x + 2)^2 - 2$
(b) minimum value -2 when $x = -2$
(c) $3(x + 2)^2 = 2$, so $(x + 2)^2 = \frac{2}{3}$

$x + 2 = \pm\sqrt{\frac{2}{3}}$

Answer: (a) $3(x + 2)^2 - 2$ (b) minimum -2 at $x = -2$ (c) $x = -2 \pm \frac{\sqrt{6}}{3}$

3 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 52 and standard deviation 3.
A value is treated as an outlier if it is more than 3 standard deviations from the mean.
Which of 39, 45, 64, 70 are outliers?

3 standard deviations = $3 \times 3 = 9$
Lower limit = $52 - 9 = 43$
Upper limit = $52 + 9 = 61$
Outliers are values below 43 or above 61
Answer: 39, 64, 70

4 Algebraic fractions (All boards)

Express $\frac{1}{(x + 1)} - \frac{2}{(x + 3)}$ as a single fraction in its simplest form.

Common denominator $(x + 1)(x + 3)$
Numerator: $1(x + 3) - 2(x + 1) = -x + 1$
Answer: $\frac{-x + 1}{(x + 1)(x + 3)}$

5 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that
$\frac{\sin 2\theta \tan \theta}{1 - \cos 3\theta}$
approaches as $\theta \to 0$.

$\sin 2\theta \approx 2\theta$, $\tan \theta \approx \theta$, $1 - \cos 3\theta \approx \frac{1}{2}(3\theta)^2$

Expression $\approx \frac{2\theta \times \theta}{\frac{9}{2}\theta^2} = \frac{2\theta^2}{\frac{9}{2}\theta^2} = \frac{4}{9}$

Answer: $\frac{4}{9}$

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6 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 6 m, mass 12 kg) has its centre of mass at G, AG = 1 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 5.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (12 + m)g$, so $T = \frac{1}{2}(12 + m)g$
Moments about A: $T \times 6 = 12g \times 1 + mg \times 5.5$
$\frac{1}{2}(12 + m) \times 6 = 12 + 5.5m \rightarrow 36 + 3m = 12 + 5.5m$
$m(5.5 - 3) = 36 - 12$, so $m = 9.6$
(b) $T = \frac{1}{2}(12 + 9.6) \times 9.8$
Answer: (a) m = 9.6 (b) 105.84 N

7 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = e^{2x}$, the x-axis, $x = 0$ and $x = 2$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = e^{4x}$, so $V = \pi\int_0^2 y^2\,dx$

$= \pi \times \frac{e^8 - 1}{4}$

$V = \frac{\pi e^8 - \pi}{4}$

Answer: $\boldsymbol{\frac{\pi e^8 - \pi}{4}}$

8 Asymptotes after transformation (All boards)

The curve $y = f(x)$, where $f(x) = \frac{3}{x+2} - 2$, is translated by $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of $y = f(x)$: $x = -2$, $y = -2$

Translate by $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$: $x = 1$, $y = -3$

New curve: $y = \frac{3}{x-1} - 3$

$y = 0$: $\frac{3}{x-1} = 3$, so $x - 1 = 1$

Answer: x = 1, y = −3; crosses x-axis at (2, 0)

9 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (2\sin 2x + 6\cos x)\,dx$.

Integrate: $[-\cos 2x + 6\sin x]$
Substitute the limits and subtract: 8
Answer: 8

10 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$, and passes through A, 30 m horizontally from O and 20.4 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $30 = U\cos\alpha \times t$, so $t = 30 \div (0.6\,U)$
Vertical: $20.4 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$20.4 = 40 - 4.9 \times 30^2 \div (0.36\,U^2)$
$U^2 = 12250 \div 19.6 = 625$, $U = 25$
Answer: 25 m s^{-1}