**Day 224** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 values have $\Sigma x = 230$ and $\Sigma x^2 = 5930$.
One value was recorded as 47 but should have been 30.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 230 - 47 + 30 = 213$

$\Sigma x^2 = 5930 - 47^2 + 30^2 = 4621$

Mean $= \frac{213}{10} = 21.3$

sd $= \sqrt{\frac{4621}{10} - (21.3)^2} = 2.90$

**Answer: mean 21.3, sd 2.90**

## 2 Implicit differentiation: products and cubes (All boards)

A curve has equation $x^2y + 2y^3 + 2x = 20$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at $(-2, 2)$.

Differentiate: $2xy + x^2\frac{dy}{dx} + 6y^2\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{2xy + 2}{x^2 + 6y^2}$

At $(-2, 2)$: $\frac{dy}{dx} = \frac{3}{14}$, so the normal has gradient $-\frac{14}{3}$

**Answer: $\frac{dy}{dx} = -\frac{2xy + 2}{x^2 + 6y^2}$; normal gradient $-\frac{14}{3}$**

## 3 Algebraic fractions (All boards)

Simplify $\frac{x^2 - 16}{(x - 3)} \div \frac{(x + 4)}{(3x - 1)}$.

$\frac{x^2 - 16}{(x - 3)} \times \frac{(3x - 1)}{(x + 4)}$

$x^2 - 16 = (x - 4)(x + 4)$

Cancel $(x + 4)$.

**Answer: $\frac{(x - 4)(3x - 1)}{(x - 3)}$**

## 4 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (6\mathbf{i} - 2\mathbf{j})$ N, $F_2 = (24\mathbf{i} + 7\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 5 kg particle in equilibrium. Find (a) a and b. $F_2$ is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

**i**: $6 + 24 + a = 0$, so $a = -30$

**j**: $-2 + 7 + b = 0$, so $b = -5$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(24^2 + 7^2)} = 25$ N

$a = 25 \div 5$

**Answer: (a) $a = -30$, $b = -5$ (b) 5 m s$^{-2}$**

## 5 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 + 2$ and the line $y = 2x + 2$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 + 2 = 2x + 2$: $x^2 - 2x = 0$

$x(x - 2) = 0$, $x = 0$ or 2

A(0, 2), B(2, 6)

Midpoint: average the coordinates

**Answer: (a) A(0, 2), B(2, 6) (b) (1, 4)**

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## 6 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 8 kg box is pulled along rough horizontal ground, $\mu = 0.1$, by a rope at angle $\theta$ above the horizontal, where $\tan\theta = \frac{3}{4}$. The tension is 35 N. Find (a) the normal reaction, (b) the acceleration of the box. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\theta = 0.6$, $\cos\theta = 0.8$
(a) Vertical: $R + 35\sin\theta = 8g$, so $R = 57.4$
(b) Horizontal: $35\cos\theta - \mu R = 8a$
$28 - 0.1\times57.4 = 8a$
$a = 2.7825$

**Answer: (a) 57.4 N (b) 2.78 m s$^{-2}$**

## 7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of $x^{-2}$ in the expansion of $(3x^2 + \frac{3}{x^2})^7$.

General term: ${}^7C_r\,(3x^2)^{7-r}\,(3x^{-2})^r$
Power of x: $2(7 - r) - 2r = -2$
$14 - 4r = -2$ so $r = 4$
Coefficient = ${}^7C_4 \times 3^3 \times 3^4 = 76545$

**Answer: 76545**

## 8 Partial fractions and integration (All boards)

(a) Express $\frac{-2x - 9}{(x + 4)(x + 3)}$ in partial fractions.

(b) Hence find $\int_0^1 \frac{-2x - 9}{(x + 4)(x + 3)}\,dx$, giving the answer in the form ln k.

$-2x - 9 \equiv A(x + 3) + B(x + 4)$: $x = -3$ gives $A = 1$; $x = -4$ gives $B = -3$
$I = [\ln(x + 4) - 3\ln(x + 3)]_0^1$
$= \ln\frac{5}{4} - 3\ln\frac{4}{3}$
$= \ln\frac{135}{256}$

**Answer: (a) $\frac{1}{x + 4} - \frac{3}{x + 3}$; (b) $\ln\frac{135}{256}$**

## 9 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{4}{625}x^3$ for $0 \le x \le 5$, and 0 otherwise.
Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^5 x\,f(x)\,dx = 4$
$E(X^2) = \int_0^5 x^2\,f(x)\,dx = \frac{50}{3}$
$\text{Var}(X) = \frac{50}{3} - (4)^2 = \frac{2}{3}$
Standard deviation $= \sqrt{\frac{2}{3}} = 0.816$

**Answer: E(X) = 4, Var(X) = $\frac{2}{3}$, sd 0.816**

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## 10 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $2x^2 + 4xy - 3x - 3y = 19$.
Show that P$(-1, -2)$ lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(4x + 4y - 3) + (4x - 3)\frac{dy}{dx} = 0$

At P$(-1, -2)$: $-15 + (-7)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{15}{7}$

Tangent: $y + 2 = (-\frac{15}{7})(x + 1)$, i.e. $15x + 7y + 29 = 0$

**Answer: $15x + 7y + 29 = 0$**