Day 222 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Combined inequalities (All boards)

(a) Solve $4x - 8 \leq 5$.
(b) Solve $x^2 - 36 > 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $4x - 8 \leq 5$ so $x \leq \frac{13}{4}$
(b) $(x + 6)(x - 6) > 0$: critical values -6 and 6
(b) gives $x < -6$ or $x > 6$
(c) Overlap of the two sets.
Answer: (a) $x \leq \frac{13}{4}$ (b) $x < -6$ or $x > 6$ (c) $x < -6$

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m s^{-1} and accelerates uniformly. She travels 127.5 m in the next 15 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $127.5 = 4\times15 + \frac{1}{2}\times a\times15^2$, so $a = 0.6$
(b) $v = u + at = 4 + 0.6\times15 = 13$
(c) For the midpoint, $s = 63.75$: $v^2 = u^2 + 2as = 4^2 + 2\times0.6\times63.75$
$v^2 = 92.5$
Answer: (a) 0.6 m s^{-2} (b) 13 m s^{-1} (c) 9.62 m s^{-1}

3 Chain rule and gradients (All boards)

$y = (\ln x)^3$ for $x > 0$.
Find $\frac{dy}{dx}$ and its exact value at $x = e^2$.

Let $u = \ln x$: $y = u^3$, so $\frac{dy}{dx} = 3u^2 \times \frac{1}{x} = \frac{3(\ln x)^2}{x}$
At $x = e^2$: $\ln x = 2$, so gradient $= \frac{12}{e^2}$
Answer: $\frac{dy}{dx} = \frac{3(\ln x)^2}{x}$; gradient $= \frac{12}{e^2}$

4 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(2 - 3x)^{-2} = 2^{-2}(1 + \frac{-3x}{2})^{-2}$
$2^{-2} = \frac{1}{4}$
Expand $(1 + y)^{-2}$ with $y = \frac{-3x}{2}$
Valid for $|\frac{-3x}{2}| < 1$, i.e. $|x| < \frac{2}{3}$
Answer: $\frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2$; $|x| < \frac{2}{3}$

5 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D\times2.5 = 40\ g\times0.5 - 4\ g\times2$
$R_D = 47.0$
Resolving vertically: $R_C + R_D = 44\ g = 431.2$
$R_C = 431.2 - 47.0$
Answer: $R_C = 384$ N, $R_D = 47.04$ N

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6 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{3}{x-3} + 2$, is translated by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = 3, y = 2

Translate by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$: x = 0, y = 3

New curve: y = $\frac{3}{x} + 3$

y = 0: x = -1

Answer: x = 0, y = 3; crosses x-axis at $(-1, 0)$

7 Independent and mutually exclusive events (All boards)

P(A) = 0.4 and P(A ∪ B) = 0.76.

(a) Given that A and B are independent, find P(B).

(b) Given instead that A and B are mutually exclusive, find P(B).

(c) In the case in (a), find P(A′ ∩ B).

(a) P(A ∪ B) = P(A) + P(B) − P(A)P(B), so 0.76 = 0.4 + P(B)(1 − 0.4)

P(B) = $\frac{0.76 - 0.4}{0.6}$ = 0.6

(b) P(A ∩ B) = 0, so P(B) = 0.76 − 0.4 = 0.36

(c) P(A′ ∩ B) = P(A′)P(B) = 0.6 × 0.6 = 0.36

Answer: (a) 0.6 (b) 0.36 (c) 0.36

8 Differential equations in context (All boards)

A drink at 88°C is left in a room at 18°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 18)$.

After 15 minutes the temperature is 58°C. Find k, then the time for it to reach 33°C, to 3 s.f.

$\int \frac{1}{\theta - 18}\, d\theta = \int -k\, dt$, so $\ln(\theta - 18) = -kt + C$

t = 0: θ − 18 = 70, so $\theta - 18 = 70e^{-kt}$

t = 15: $40 = 70e^{-15k}$, so k = $\frac{\ln(\frac{7}{4})}{15}$ = 0.0373

$15 = 70e^{-kt}$: t = $\frac{\ln(\frac{14}{3})}{k}$ = 41.3 minutes

Answer: k = 0.0373, t = 41.3 minutes

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(26, p). H_0: p = 0.5 is tested against H_1: p > 0.5 using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when p = 0.7 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: X ≥ 18

(b) Type I error = P(reject H_0 | p = 0.5) = 0.0378

(c) Type II error = P(not in critical region | p = 0.7)

P(X ≤ 17) when p = 0.7: 0.373

Answer: (a) X ≥ 18 (b) 0.0378 (c) 0.373

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10 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{8}{17}$.

Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos\theta = -\sqrt{1 - \left(\frac{8}{17}\right)^2} = -\frac{15}{17}$ (obtuse)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{8}{17} \times \left(-\frac{15}{17}\right) = -\frac{240}{289}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \left(\frac{8}{17}\right)^2 = \frac{161}{289}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{240}{161}$

Answer: $\sin 2\theta = -\frac{240}{289}$, $\cos 2\theta = \frac{161}{289}$, $\tan 2\theta = -\frac{240}{161}$