Day 221 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.8, cos α = 0.6; friction F = μR = 0.2×3 g cos α
B: 7 g − T = 7 a
A: T − 3 g sin α − F = 3 a
Adding: g(7 − 2.4 − 0.36) = 10 a
(a) a = 4.1552; (b) T = 7(g − a)
Answer: (a) 4.16 m s^{-2} (b) 39.5 N

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^1 (3x - 3)^2 \, dx$.

Integrate: $[\frac{(3x - 3)^3}{9}]$

Substitute the limits and subtract: 3
Answer: 3

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is normally distributed with mean 171 and standard deviation 7.
Find (a) P(X > 186.4) (b) P(186.4 < X < 190.6), each to 3 s.f.

(a) $z = \frac{186.4 - 171}{7} = 2.20$, P(Z > 2.20) = 0.0139

(b) $z_1 = 2.20$, $z_2 = \frac{190.6 - 171}{7} = 2.80$

P(186.4 < X < 190.6) = Φ(2.80) − Φ(2.20) = 0.0113
Answer: (a) 0.0139 (b) 0.0113

4 Perpendicular bisector and area (All boards)

A is (−2, −2) and B is (−6, 6).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (−4, 2); gradient AB = −2

Perpendicular gradient = $\frac{1}{2}$

Equation: x − 2y = −8
y = 0: P = (−8, 0)
Area = ½ |(−6 − (−2))(0 − (−2)) − (−8 − (−2))(6 − (−2))|
Answer: (a) x − 2y = −8 (b) 20

5 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 3 m, mass 4 kg) has its centre of mass at G, AG = 1.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 2.75 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: 2T = (4 + m)g, so T = ½(4 + m)g
Moments about A: T×3 = 4 g×1.25 + m g×2.75
½(4 + m)×3 = 5 + 2.75 m → 6 + 1.5 m = 5 + 2.75 m
m (2.75 − 1.5) = 6 − 5, so m = 0.8
(b) T = ½(4 + 0.8)×9.8
Answer: (a) m = 0.8 (b) 23.52 N

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6 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (−1, 5, 1), (−1, 5, −3) and (−3, −3, −2).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix}0\\0\\4\end{pmatrix}$, $\text{BC} = \begin{pmatrix}-2\\-8\\1\end{pmatrix}$

BA·BC = 4

$|\text{BA}| = \sqrt{16}$, $|\text{BC}| = \sqrt{69}$

$\cos \text{ABC} = \frac{4}{\sqrt{16} \times \sqrt{69}} = 0.1204$, so angle ABC = 83.1°

Answer: 83.1°

7 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.
Find the coordinates of the turning points of
(a) y = f(x + 3) (b) y = f(x) − 1

(a) y = f(x + 3): A → (−5, −1), B → (−3, 3)
(b) y = f(x) − 1: A → (−2, −2), B → (0, 2)
Answer: (a) (−5, −1), (−3, 3); (b) (−2, −2), (0, 2)

8 Normal approximation to the binomial (All boards)

X ~ B(60, 0.3). Use a normal approximation to estimate P(X ≥ 21) to 3 significant figures.

np = 18, npq = 12.6, so Y ~ N(18, 12.6)
With continuity correction: P(Y > 20.5)
$z = \frac{20.5 - 18}{3.55} = 0.704$, P(Z > 0.704) = 0.241
Answer: 0.241

9 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y + 2y^3 - 4x = -22$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (2, −1).

Differentiate: $6xy + 3x^2\frac{dy}{dx} + 6y^2\frac{dy}{dx} - 4 = 0$

$\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 + 6y^2}$

At (2, −1): $\frac{dy}{dx} = \frac{8}{9}$, so the normal has gradient $-\frac{9}{8}$

Answer: $\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 + 6y^2}$; normal gradient $-\frac{9}{8}$

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-12x + 4}{(1 + 2x)(1 - 2x)}$.
(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 2x)} + \frac{B}{(1 - 2x)}$: A = 5, B = −1
(b) $5(1 + 2x)^{-1} = 5(1 - 2x + 4x^2 \ldots)$
Add the series for both fractions.
(c) Need |2x| < 1 and |2x| < 1
Answer: (a) $\frac{5}{(1 + 2x)} + \frac{-1}{(1 - 2x)}$ (b) $4 - 12x + 16x^2$ (c) $|x| < \frac{1}{2}$