Day 220 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + \sin x - 1 = 0$
for $0° \le x \le 360°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - \sin x - 1 = 0$

Solving the quadratic: $\sin x = -\frac{1}{2}$ or $\sin x = 1$

$\sin x = -\frac{1}{2}$: $x = 210°, 330°$

$\sin x = 1$: $x = 90°$

Answer: $x = 90°, 210°, 330°$

2 Polynomial division (All boards)

Find the quotient and the remainder when $2x^3 + 14x^2 + 21x + 11$ is divided by $(x + 5)$.

Quotient (matching leading terms in turn): $2x^2 + 4x + 1$

$(x + 5)(2x^2 + 4x + 1) = 2x^3 + 14x^2 + 21x + 5$

Remainder $= 2x^3 + 14x^2 + 21x + 11 - (2x^3 + 14x^2 + 21x + 5) = 6$

Answer: quotient $2x^2 + 4x + 1$, remainder 6

3 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily mean temperature in August is modelled by $N(16, 2.6^2)$.
(a) Find the probability that on a randomly chosen day the value exceeds 20 °C.
(b) Estimate the number of such days in a period of 92 days.

(a) $z = \frac{20 - 16}{2.6} = 1.54$

$P(Z > 1.54) = 0.0620$

(b) $92 \times 0.0620 = 5.70 \approx 6$

Answer: (a) 0.0620 (b) 6 days

4 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 20 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.25$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take $g = 9.8$.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: $R = 20g + P\sin\theta$

Horizontal: $P\cos\theta = \mu R = 0.25(20g + P\sin\theta)$

$P(\frac{5}{13} - \frac{3}{13}) = 0.25 \times 20g$

$P = 319$, $R = 490$

Answer: P = 318.5 N, R = 490 N

5 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 6x + 14$, $x \ge 3$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = (x - 3)^2 + 5$

Let $y = f(x)$: $(x - 3)^2 = y - 5$

$x \ge 3$, so take the positive root: $x = 3 + \sqrt{y - 5}$

Answer: $f^{-1}(x) = 3 + \sqrt{x - 5}$, $x \ge 5$

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6 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -3 \end{pmatrix}$ and $\begin{pmatrix} k \\ -2 \\ 1 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - 2k - 3 = 0$

$(k - 3)(k + 1) = 0$

$k = -1$ or $k = 3$. For $k = 3$ the first vector is $\begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$, with length $\sqrt{27} = 3\sqrt{3}$

Answer: $k = -1$ or 3; length $3\sqrt{3}$

7 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve $y = f(x)$, where $f(x)$ is a cubic.

Find $f(x)$, giving your answer in the form $k(x - p)(x - q)^2$.

Crosses at $x = 2$ and touches at $x = -1$, so $f(x) = k(x - 2)(x + 1)^2$

$f(0) = -4$: $k(-2)(1)^2 = -4$

$k = 2$

Answer: $f(x) = 2(x - 2)(x + 1)^2$

8 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(35, p)$. H_0: $p = 0.3$ is tested against H_1: $p \neq 0.3$ at the 10% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.05. $P(X \leq 5) = 0.0269$, $P(X \leq 6) = 0.0650$

$P(X \geq 16) = 0.0359$, $P(X \geq 15) = 0.0731$

Critical region: $X \leq 5$ or $X \geq 16$

(b) Actual level = 0.0269 + 0.0359 = 0.0628

Answer: (a) $X \leq 5$ or $X \geq 16$ (b) 0.0628

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + 4y^2 = 14$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 8y}$

Horizontal tangent: $4x + 2y = 0$, so $y = -2x$

Substitute: $14x^2 = 14$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, -2)$ and $(-1, 2)$

Answer: $(1, -2)$ and $(-1, 2)$

10 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 4.2 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{12}{5}$. The coefficient of friction is 0.6. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Up: deceleration $= g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{12}{13} + 0.6\times\frac{5}{13}) = \frac{147}{13}$

(a) $0 = 4.2^2 - 2\times\frac{147}{13}\times d$, so $d = 0.78$

Down: $\tan\alpha = \frac{12}{5} > \mu$, so it slides back; acceleration $= g(\sin\alpha - \mu\cos\alpha) = \frac{441}{65}$

(b) $v^2 = 2\times\frac{441}{65}\times 0.78 = 10.584$

$v = \sqrt{10.584}$

Answer: (a) 0.78 m (b) 3.25 m s^{-1}