Day 22 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(62, 10^2)$. Find $P(X > 84 \mid X > 69)$ to 3 significant figures.

Since $X > 84$ implies $X > 69$: $P = \frac{P(X > 84)}{P(X > 69)}$

$P(X > 84) = P(Z > 2.20) = 0.0139$

$P(X > 69) = P(Z > 0.700) = 0.242$

$P = \frac{0.0139}{0.242} = 0.0575$

Answer: 0.0575

2 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−1, 0) and passes through B(−5, 2).
Find the equation of the circle.

Touches the x-axis at A, so the centre is $(-1, r)$ and the radius is r.

B on the circle: $(-4)^2 + (2 - r)^2 = r^2$

$16 + 4 - 4r = 0$ so $r = 5$

Answer: $(x + 1)^2 + (y - 5)^2 = 25$

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 22 pairs of values of time spent on social media and exam score, $r = 0.292$.
The critical value for a one-tailed test at the 5% level is 0.3598.
Test, at this level, whether there is positive correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho > 0$

Critical value 0.3598

$r = 0.292 < 0.3598$

This is not in the critical region, so do not reject H_0.

There is insufficient evidence of correlation.

Answer: Do not reject H_0: insufficient evidence

4 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.25$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take $g = 9.8$.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, weight $W = 39.2$ N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 31.9$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 93.1$

Answer: least 31.85 N, greatest 93.1 N

5 Line meets curve: length of chord (All boards)

The line $y = x + 2$ meets the curve $y = x^2 + 2$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 + 2 = x + 2$ gives $x^2 - x = 0$

$x(x - 1) = 0$: $x = 0$ or 1

A(0, 2), B(1, 3)

$AB^2 = 1^2 + 1^2 = 2$

Answer: A(0, 2), B(1, 3); AB = $\sqrt{2}$

6 Differentiating sec, cosec, tan and cot (All boards)

y = cot(3x).

Find $\frac{dy}{dx}$ and its exact value at x = $\frac{\pi}{9}$.

$\frac{dy}{dx} = -3\text{cosec}^2(3x)$

At x = $\frac{\pi}{9}$ the angle 3x = $\frac{\pi}{3}$, so the gradient is -4

Answer: $\mathbf{\frac{dy}{dx} = -3\text{cosec}^2(3x);\ -4}$

7 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 12.6 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where tan α = $\frac{12}{5}$. The coefficient of friction is 0.6. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration = g(sin α + μ cos α) = $9.8(\frac{12}{13} + 0.6\times\frac{5}{13}) = \frac{147}{13}$

(a) $0 = 12.6^2 - 2\times\frac{147}{13}\times d$, so d = 7.02

Down: tan α = $\frac{12}{5}$ > μ, so it slides back; acceleration = g(sin α − μ cos α) = $\frac{441}{65}$

(b) $v^2 = 2\times\frac{441}{65}\times 7.02 = 95.256$

$v = \sqrt{95.256}$

Answer: (a) 7.02 m (b) 9.76 m s^{-1}

8 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y + y^3 + 4x = 40$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (2, 2).

Differentiate: $6xy + 3x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} + 4 = 0$

$\frac{dy}{dx} = -\frac{6xy + 4}{3x^2 + 3y^2}$

At (2, 2): $\frac{dy}{dx} = -\frac{7}{6}$, so the normal has gradient $\frac{6}{7}$

Answer: $\mathbf{\frac{dy}{dx} = -\frac{6xy + 4}{3x^2 + 3y^2};}$ **normal gradient** $\mathbf{\frac{6}{7}}$

9 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 12 and its sum to infinity is 54.
Find the two possible values of the first term.

ar = 12 and $\frac{a}{(1 - r)} = 54$

a = 54(1 − r), so 54(1 − r) r = 12

$54r^2 - 54r + 12 = 0$: r = $\frac{1}{3}$ or r = $\frac{2}{3}$

a = 54(1 − r) = 36 or 18

Answer: a = 18 or a = 36

10 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta - 10\tan\theta + 10 = 0$
for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta - 10\tan\theta + 12 = 0$
$\tan\theta = 2$ or $\tan\theta = 3$
$\tan\theta = 2$: $\theta = 63.4°, 243.4°$
$\tan\theta = 3$: $\theta = 71.6°, 251.6°$
Answer: θ = 63.4°, 71.6°, 243.4°, 251.6°