Day 219 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Perpendicular bisector and area (All boards)

A is (5, −5) and B is (1, 1).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (3, −2); gradient AB = $-\frac{3}{2}$

Perpendicular gradient = $\frac{2}{3}$

Equation: 2x − 3y = 12
y = 0: P = (6, 0)
Area = ½ |(1 − 5)(0 − (−5)) − (6 − 5)(1 − (−5))|
Answer: (a) 2x − 3y = 12 (b) 13

2 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 3x}{x \sin x}$ approaches as x → 0.

$1 - \cos 3x \approx \frac{1}{2}(3x)^2 = \frac{9}{2}x^2$

$\sin x \approx x$

Expression $\approx \frac{\frac{9}{2}x^2}{x \times x} = \frac{9}{2} = \frac{9}{2}$

Answer: $\frac{9}{2}$

3 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $25^{3x-1} = 5^{2x-2}$.

Write both sides as powers of 5.
$(5^2)^{3x-1} = (5^1)^{2x-2}$
2(3x − 1) = 2x − 2
6x − 2 = 2x − 2
4x = 0
Answer: x = 0

4 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 6 kg box is pulled from rest along rough horizontal ground by a horizontal force of 19.44 N and moves 15 m in 10 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $15 = \frac{1}{2}a \times 10^2$, so a = 0.3
(a) P − μ6 g = 6 a: 19.44 − 58.8 μ = 1.8, so μ = 0.3
(b) v = at = 0.3×10 = 3; after removal deceleration = μg = 2.94
$0 = 3^2 - 2 \times 2.94 \times s$, s = 1.53
Answer: (a) μ = 0.3 (b) 1.53 m

5 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(2, 0) and passes through B(6, 4).
Find the equation of the circle.

Touches the x-axis at A, so the centre is (2, r) and the radius is r.
B on the circle: $(4)^2 + (4 - r)^2 = r^2$
16 + 16 − 8r = 0 so r = 4
Answer: $(x - 2)^2 + (y - 4)^2 = 16$

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6 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(6t - 6)$ m s^{-2}. When t = 0 it is at O with velocity -7 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 1

$v = \int a\,dt = 3t^2 - 6t - 7$ (using v = −7 when t = 0)

v(1) = −10

$x = \int v\,dt = t^3 - 3t^2 - 7t$ (x = 0 when t = 0)

x(1) = −9

Answer: (a) −10 m s^{-1} (b) −9 m

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The length of a plank is N(60, 3^2). A plank is classed as 'special' if its length is more than 63. 12 planks are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X > 63) = P(Z > 1.00) = 0.159

Y ~ B(12, 0.159)

$P(Y = 3) = C(12, 3)(0.159)^3(0.841)^9 = 0.186$

Answer: 0.186

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$. Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

$n = \frac{7 - 4}{4 - 2} = \frac{3}{2}$

At (2, 4): $\ln A = 4 - \frac{3}{2} \times 2 = 1$, so $A = e^1$

Answer: n = $\frac{3}{2}$, A = e^1

9 Partial fractions and integration (All boards)

(a) Express $\frac{-2x - 7}{(x + 3)^2}$ in the form $\frac{A}{x + 3} + \frac{B}{(x + 3)^2}$.

(b) Hence find the exact value of $\int_0^3 \frac{-2x - 7}{(x + 3)^2}\,dx$.

$-2x - 7 \equiv A(x + 3) + B$, so A = −2 and B = −1

$I = [-2 \ln(x + 3) - \frac{-1}{x + 3}]_0^3$

$= -2 \ln \frac{6}{3} + -1(\frac{1}{3} - \frac{1}{6})$

$= \frac{-12 \ln 2 - 1}{6}$

Answer: (a) A = −2, B = −1; (b) $\frac{-12 \ln 2 - 1}{6}$

10 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(36, p). H_0: p = 0.4 is tested against H_1: p ≠ 0.4 at the 5% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. P(X ≤ 8) = 0.0196, P(X ≤ 9) = 0.0449

P(X ≥ 21) = 0.0200, P(X ≥ 20) = 0.0426

Critical region: X ≤ 8 or X ≥ 21

(b) Actual level = 0.0196 + 0.0200 = 0.0396

Answer: (a) X ≤ 8 or X ≥ 21 (b) 0.0396