Day 218 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg carries a person of mass 75 kg. The cable tension is 3250 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 325 kg), taking upwards as positive: $T - 325g = 325a$

$3250 - 3185 = 325a$, so $a = 0.2$

(a) a is positive, so the acceleration is 0.2 m s^{-2} upwards

(b) Person: $R - 75g = 75a$, so $R = 75(9.8 + 0.2)$

Answer: (a) 0.2 m s^{-2} upwards (b) 750 N

2 Partial fractions: repeated factor (All boards)

Express $\frac{-2x^2 - 6x + 5}{(x+2)(x-1)^2}$ in the form $\frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$, and find A, B and C.

$-2x^2 - 6x + 5 \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$

$x = -2$: $9 = A(-3)^2$ so $A = 1$

$x = 1$: $-3 = C(3)$ so $C = -1$

Coefficient of x^2: $-2 = A + B$ so $B = -3$

Answer: A = 1, B = −3, C = −1

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is normally distributed with mean 74 and standard deviation 12.

Find (a) P(X > 64.4) (b) P(64.4 < X < 82.4), each to 3 s.f.

(a) $z = \frac{64.4 - 74}{12} = -0.800$, $P(Z > -0.800) = 0.788$

(b) $z_1 = -0.800$, $z_2 = \frac{82.4 - 74}{12} = 0.700$

$P(64.4 < X < 82.4) = \Phi(0.700) - \Phi(-0.800) = 0.546$

Answer: (a) 0.788 (b) 0.546

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$OP = \sqrt{3^2 + (-1)^2} = \sqrt{10} = \sqrt{10}$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{-1} \end{pmatrix}$**, OP =** $\sqrt{\mathbf{10}}$

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 - 5x + 3$ is transformed by a stretch, scale factor 2, parallel to the x-axis, followed by a stretch, scale factor 2, parallel to the y-axis.

Find the equation of the image curve.

Stretch parallel to x: replace x by $\frac{x}{2}$: $y = \frac{1}{4}x^2 - \frac{5}{2}x + 3$

Stretch parallel to y: multiply by 2: $y = \frac{1}{2}x^2 - 5x + 6$

Answer: $\mathbf{y = \frac{1}{2}x^2 - 5x + 6}$

6 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 107.8 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.
$4.9t^2 - 4.9(t - 2)^2 = 107.8$
$4.9(4t - 4) = 107.8$, so $t = 6$
Height $= 4.9 \times 6^2 = 207.025$
Answer: 207.025 m

7 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is $N(63, 5^2)$.
Find the symmetric limits about the mean between which the central 90% of values lie (3 s.f.).

Each tail has area 0.05, so $P(Z < z) = 0.95$
$z = 1.64$
Limits: $63 \pm 1.64 \times 5$
$= 54.8$ and 71.2
Answer: 54.8 to 71.2

8 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a\left(\frac{4}{\sqrt{3}}\right)$ in terms of p and q.

$\frac{4}{\sqrt{3}} = 2^2 \times 3^{-\frac{1}{2}}$

$\log_a(2^2 \times 3^{-\frac{1}{2}}) = 2\log_a 2 - \frac{1}{2}\log_a 3$

$= 2p - \frac{1}{2}q$

Answer: $2p - \frac{1}{2}q$

9 Implicit differentiation: products and cubes (All boards)

A curve has equation $2x^2y + 2y^3 + 3x = 16$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (2, 1).

Differentiate: $4xy + 2x^2\frac{dy}{dx} + 6y^2\frac{dy}{dx} + 3 = 0$

$\frac{dy}{dx} = -\frac{4xy + 3}{2x^2 + 6y^2}$

At (2, 1): $\frac{dy}{dx} = -\frac{11}{14}$, so the normal has gradient $\frac{14}{11}$

Answer: $\frac{dy}{dx} = -\frac{4xy + 3}{2x^2 + 6y^2}$; normal gradient $\frac{14}{11}$

10 Factor theorem and remainder theorem (All boards)

$f(x) = x^3 + ax^2 + bx - 32$, where a and b are constants.
$(x + 2)$ is a factor of f(x). The remainder when f(x) is divided by $(x + 1)$ is -15.
(a) Find a and b. (b) Solve $f(x) = 0$.

$f(-2) = 0$: $-8 + 4a - 2b - 32 = 0$
$f(-1) = -15$: $-1 + a - b - 32 = -15$
Solving simultaneously: $a = 2$, $b = -16$
$f(x) = x^3 + 2x^2 - 16x - 32 = (x + 2)(x + 4)(x - 4)$
Answer: (a) a = 2, b = −16 (b) x = −4, −2, 4