Day 217 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - 6x + 2$ and the line $y = -2x - 1$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 - 6x + 2 = -2x - 1$: $x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$, $x = 1$ or 3
A(1, −3), B(3, −7)
Midpoint: average the coordinates
Answer: (a) A(1, −3), B(3, −7) (b) (2, −5)

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A club has 130 members: 60 Bronze, 30 Silver and 40 Gold.
A stratified sample of size 26 is taken.
Find the number of members sampled from each group.

Sampling fraction = $\frac{26}{130} = \frac{1}{5}$

Bronze: $60 \times \frac{1}{5} = 12$

Silver: $30 \times \frac{1}{5} = 6$

Gold: $40 \times \frac{1}{5} = 8$

Answer: Bronze 12, Silver 6, Gold 8

3 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 10 kg box is pulled from rest along rough horizontal ground by a horizontal force of 15.8 N and moves 19.2 m in 8 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $19.2 = \frac{1}{2}a \times 8^2$, so $a = 0.6$
(a) $P - \mu 10\,g = 10\,a$: $15.8 - 98\,\mu = 6$, so $\mu = 0.1$
(b) $v = at = 0.6 \times 8 = 4.8$; after removal deceleration = $\mu g = 0.98$
$0 = 4.8^2 - 2 \times 0.98 \times s$, $s = 11.8$
Answer: (a) μ = 0.1 (b) 11.8 m

4 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,
express $\log_a(24)$ in terms of p and q.

$24 = 2^3 \times 3$
$\log_a(2^3 \times 3) = 3\log_a 2 + \log_a 3$
$= 3p + q$
Answer: 3p + q

5 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |2x + 3|$ and $y = x + 6$.
The graphs meet at A and B.
Find the coordinates of A and B.

$|2x + 3| = x + 6$
Solve $2x + 3 = \pm(x + 6)$.
$x = -3$ and $x = 3$ (both checked in the original equation)
$y = 3$ and $y = 9$
Answer: A(−3, 3), B(3, 9)

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6 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = 4x$,

given that y = 3 when x = 0 (y > 0). Give y in terms of x.

Separate: $\int y\,dy = \int 4x\,dx$, so $\frac{y^2}{2} = 2x^2 + C$

x = 0, y = 3 gives $C = \frac{9}{2}$

$y^2 = 4x^2 + 9$

Answer: $y = \sqrt{4x^2 + 9}$

7 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses in grams of two groups: factory A has n = 13, mean 69, sd 6;
factory B has n = 13, mean 57, sd 5.
Find the mean and standard deviation of all 26 values (3 s.f. for the sd).

$\Sigma x = 13 \times 69 + 13 \times 57 = 1638$, mean = 63

Σx^2 (A) $= 13(6^2 + 69^2) = 62361$

Σx^2 (B) $= 13(5^2 + 57^2) = 42562$

Variance $= \frac{104923}{26} - 63^2 = 66.5$, so sd = 8.15

Answer: mean 63, sd 8.15

8 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(11, −2) and B(7, −10).
Its centre lies on the line $y = -x - 1$.
Find the equation of the circle.

Centre $(h, -h - 1)$ is equidistant from A and B.

$(h - 11)^2 + (-h + 1)^2 = (h - 7)^2 + (-h + 9)^2$

Solving gives h = 1, so centre (1, −2)

$r^2 = (10)^2 + (0)^2 = 100$

Answer: $(x - 1)^2 + (y + 2)^2 = 100$

9 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 4 m s^{-1}. 4 s later car B leaves P from rest, in the same direction, with acceleration 1.5 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 4) seconds.

B: $\frac{1}{2} \times 1.5 \times s^2 = 0.75\,s^2$ A: $4(s + 4)$

$0.75\,s^2 - 4\,s - 16 = 0$ so s = 8 (s > 0)

(b) $PQ = 0.75 \times 8^2 = 48$

(c) $v = 0 + at = 1.5 \times 8 = 12$

Answer: (a) 8 s (b) 48 m (c) 12 m s^{-1}

10 Differential equations in context (All boards)

A drink at 82°C is left in a room at 22°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.

After 15 minutes the temperature is 62°C. Find k, then the time for it to reach 42°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\,d\theta = \int -k\,dt$, so $\ln(\theta - 22) = -kt + C$

t = 0: $\theta - 22 = 60$, so $\theta - 22 = 60e^{-kt}$

t = 15: $40 = 60e^{-15k}$, so $k = \frac{\ln(\frac{3}{2})}{15} = 0.0270$

$20 = 60e^{-kt}$: $t = \frac{\ln(3)}{k} = 40.6$ minutes

Answer: k = 0.0270, t = 40.6 minutes