Day 216 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 30t + 24)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 5 s.

(a) $v = 6(t - 1)(t - 4) = 0$, so $t = 1$ or $t = 4$

(b) Displacement $x = \int v\,dt = 2t^3 - 15t^2 + 24t$

$x(1) = 11$, $x(4) = -16$, $x(5) = -5$

Direction changes at $t = 1$ and $t = 4$, so distance $= |11| + |-16 - 11| + |-5 - (-16)|$

Answer: (a) $t = 1$, $t = 4$ (b) 49 m

2 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a minimum point at $(-5, 1)$.

Find the coordinates of the corresponding point on:

(a) $y = f(x - 2)$

(b) $y = 4f(x)$

(c) $y = f(\frac{x}{2})$

(a) $y = f(x - 2)$: $(-3, 1)$

(b) $y = 4f(x)$: $(-5, 4)$

(c) $y = f(\frac{x}{2})$: $(-10, 1)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) $(-3, 1)$; (b) $(-5, 4)$; (c) $(-10, 1)$

3 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-4\mathbf{i} + 6\mathbf{j})$ N, $F_2 = (9\mathbf{i} - 12\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

i: $-4 + 9 + a = 0$, so $a = -5$

j: $6 + (-12) + b = 0$, so $b = 6$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(9^2 + -12^2)} = 15$ N

$a = 15 \div 2$

Answer: (a) $a = -5$, $b = 6$ (b) 7.5 m s^{-2}

4 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 4 cm. Angle AOB = $\frac{2\pi}{3}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = \frac{16\pi}{3}$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 4\sqrt{3}$

Segment area = sector − triangle = $\frac{16\pi - 12\sqrt{3}}{3}$

Chord AB = $2r\sin\frac{\theta}{2} = 4\sqrt{3}$; arc = $r\theta = \frac{8\pi}{3}$

Perimeter = arc + chord = $\frac{12\sqrt{3} + 8\pi}{3}$

Answer: area = $\frac{16\pi - 12\sqrt{3}}{3}$ cm^2, perimeter = $\frac{12\sqrt{3} + 8\pi}{3}$ cm

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5 Arithmetic series: finding n (All boards)

An arithmetic series has first term 14 and common difference 5.
The sum of the first n terms is 1850. Find n.

$\frac{n}{2}(2 \times 14 + (n - 1) \times 5) = 1850$

$5n^2 + 23n = 3700$

$5n^2 + 23n - 3700 = 0$

$(n - 25)(5n + 148) = 0$, so $n = 25$ or $n = -\frac{148}{5}$

n must be a positive integer, so $n = 25$

Answer: 25

6 Three-set Venn diagram counts (All boards)

Of 31 students, 19 study Maths (M), 18 Physics (P) and 15 Chemistry (C).
9 study M and P, 8 study M and C, 9 study P and C, and 4 study all three.
One student is chosen at random. Given that a student studies exactly two subjects, find the probability that they study Physics.

Venn regions: M only 6, P only 4, C only 2,
MP only 5, MC only 4, PC only 5, all three 4, outside all 1
Exactly two subjects: 5 + 4 + 5 = 14
Of these, Physics: 5 + 5 = 10

Probability = $\frac{10}{14}$

Answer: $\frac{5}{7}$

7 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{4}} x \sin 2x \, dx$.

$u = x$, $\frac{dv}{dx} = \sin 2x$: $\int x \sin 2x \, dx = [-\frac{x \cos 2x}{2}] + \int \frac{\cos 2x}{2} \, dx$

$= [-\frac{x \cos 2x}{2} + \frac{\sin 2x}{4}]$

$= \frac{1}{4}$

Answer: $\frac{1}{4}$

8 Tangent conditions (All boards)

Find the set of values of k for which the line $y = -3x + k$ meets the curve $y = x^2 + x + 4$ at two distinct points.

$x^2 + x + 4 = -3x + k$ gives $x^2 + 4x + (4 - k) = 0$

Discriminant: $4^2 - 4 \times 1(4 - k) = 4k$

Two distinct points: discriminant > 0

Answer: k > 0

9 Exponential equations reducing to quadratics (All boards)

Solve $e^{2x} - 5e^x + 6 = 0$,
giving exact answers.

Let $u = e^x$: $u^2 - 5u + 6 = 0$, so $(u - 3)(u - 2) = 0$

$e^x = 3$ or $e^x = 2$

Answer: x = ln 2 or x = ln 3

10 Normal approximation to the Poisson (CIE)

X ~ Po(25). Use a suitable approximation to find P(X ≥ 21), to 3 significant figures.

$\lambda > 15$, so Y ~ N(25, 25) approximately
Continuity correction: P(Y > 20.5)
$z = -0.900$, so probability = 0.816
Answer: 0.816