Day 215 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 26.4 m ahead. After a reaction time of 1 s she brakes with deceleration 5 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 1 v, braking distance = $v^2 \div (2\times5)$

$1\,v + v^2 \div 10 = 26.4$, so $v^2 + 10\,v - 264 = 0$

$(v - 12)(v + 22) = 0$, so $v = 12$

(b) Braking time = $12 \div 5 = 2.4$ s; total = $1 + 2.4$

Answer: (a) 12 m s^{-1} (b) 3.4 s

2 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the journey time (minutes), is normally distributed with mean 35 and standard deviation 6.
Find (a) P(X > 22.4) (b) P(22.4 < X < 26.6), each to 3 s.f.

(a) $z = \frac{22.4 - 35}{6} = -2.10$, $P(Z > -2.10) = 0.982$

(b) $z_1 = -2.10$, $z_2 = \frac{26.6 - 35}{6} = -1.40$

$P(22.4 < X < 26.6) = \Phi(-1.40) - \Phi(-2.10) = 0.0629$

Answer: (a) 0.982 (b) 0.0629

3 Exact values from the addition formulae (All boards)

By writing 165° as 120° + 45°, find the exact value of

sin 165°, in the form $\frac{\sqrt{a} \pm \sqrt{b}}{c}$.

$\sin 165° = \sin(120° + 45°) = \sin 120° \cos 45° + \cos 120° \sin 45°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + (-\frac{1}{2}) \times \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} - \sqrt{2}}{4}$

4 Discriminant: range of k (All boards)

The equation $3x^2 + (k)x + (k) = 0$ has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k)^2 - 4 \times 3(k)$

$= k^2 - 12k$

Roots of the quadratic in k: k = 0 and k = 12

Required: $b^2 - 4ac < 0$

Answer: 0 < k < 12

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 3$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{3}{x_n})$,

and use it with $x_0 = \frac{5}{2}$ to find x_2 as an approximation to $\sqrt{3}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 3}{2x_n} = \frac{1}{2}(x_n + \frac{3}{x_n})$

$x_1 = \frac{1}{2}(\frac{5}{2} + \frac{3}{\frac{5}{2}}) = \frac{37}{20}$

$x_2 = 1.7358$

Answer: $x_2 = 1.7358$

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6 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-4x + 3y = k$ is a tangent to the circle
$(x + 4)^2 + (y - 4)^2 = 16$.
Find the two possible values of k.

Centre $(-4, 4)$, radius 4
Distance from centre to the line = radius:
$\frac{|28 - k|}{5} = 4$
$|28 - k| = 20$
Answer: k = 8 or k = 48

7 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 5 kg particle moves on a smooth horizontal line. At time t s a single force of $(24 - 4t)$ N acts along the line. When t = 0 its speed is 4 m s^{-1}. Find (a) its speed when t = 2, (b) its greatest speed.

F = ma: $a = (-4t + 24) \div 5$
$v = \int a\,dt = 4.8t - 0.4t^2 + c$; v = 4 when t = 0
(a) v(2) = 12
(b) Greatest speed when a = 0: t = 6
v(6) = 18.4
Answer: (a) 12 m s^{-1} (b) 18.4 m s^{-1}

8 Separable differential equations (All boards)

Solve the differential equation
$\frac{dy}{dx} = 4xy^2$,
given that y = 2 when x = 0. Give y in terms of x.

Separate: $\int \frac{1}{y^2}\,dy = \int 4x\,dx$, so $-\frac{1}{y} = 2x^2 + C$
x = 0, y = 2 gives $C = -\frac{1}{2}$
$-\frac{1}{y} = 2x^2 - \frac{1}{2}$
Answer: $y = \frac{2}{1 - 4x^2}$

9 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2 - 3$, $y = t(t^2 - 16)$.
Find the coordinates of the points where the curve meets the x-axis.

y = 0: t = 0 or $t^2 = 16$, so t = 0, 4, −4
t = 0: x = −3
t = ±4: x = 13
Answer: (−3, 0) and (13, 0)

10 Histograms (All boards)

The histogram shows the masses of apples, m (in grams), of a sample.
(a) How many were in the class $40 \leq m < 60$?
(b) Estimate how many had $20 < m < 50$.

(a) Frequency = frequency density × width = 3 × 20 = 60
(b) 20 to 25: 5 × 5 = 25
whole bar 25 to 40: 60
40 to 50: 3 × 10 = 30
Total = 115
Answer: (a) 60 (b) 115