Day 214 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = e^{2x^2 + 4x}$.

Find $\frac{dy}{dx}$, and the equation of the tangent to the curve at the point where $x = -2$.

Let $u = 2x^2 + 4x$: $\frac{dy}{dx} = \frac{du}{dx} \times e^u = (4x + 4)e^{2x^2 + 4x}$

At $x = -2$: $u = 0$, so $y = 1$ and gradient $= -4$

Tangent: $y - 1 = (-4)(x + 2)$, so $y = -4x - 7$

Answer: $\frac{dy}{dx} = (4x + 4)e^{2x^2 + 4x}$; $y = -4x - 7$

2 Polynomial division (All boards)

Find the quotient and the remainder when $3x^4 + 5x^3 - 3x^2 - 14x - 8$ is divided by $(x^2 - 2)$.

Quotient (matching leading terms in turn): $3x^2 + 5x + 3$

$(x^2 - 2)(3x^2 + 5x + 3) = 3x^4 + 5x^3 - 3x^2 - 10x - 6$

Remainder $= 3x^4 + 5x^3 - 3x^2 - 14x - 8 - (3x^4 + 5x^3 - 3x^2 - 10x - 6) = -4x - 2$

Answer: quotient $3x^2 + 5x + 3$, remainder $-4x - 2$

3 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg carries a person of mass 80 kg. The cable tension is 5974 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 580 kg), taking upwards as positive: $T - 580\,g = 580\,a$

$5974 - 5684 = 580\,a$, so $a = 0.5$

(a) a is positive, so the acceleration is 0.5 m s^{-2} upwards

(b) Person: $R - 80\,g = 80\,a$, so $R = 80(9.8 + 0.5)$

Answer: (a) 0.5 m s^{-2} upwards (b) 824 N

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{3}{4}\text{AB} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\text{OP} = \sqrt{5^2 + 0^2} = \sqrt{25} = 5$

Answer: $\begin{pmatrix} \mathbf{5} \\ \mathbf{0} \end{pmatrix}$**, OP = 5**

5 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = kx^2$ for $0 \le x \le 4$, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find $P(X > 2)$.

(a) $\int_0^4 f(x)\,dx = 1$ gives $\frac{64}{3}k = 1$, so $k = \frac{3}{64}$

(b) $E(X) = \int x\,f(x)\,dx = 3$

(c) $P(X > 2) = \int_2^4 f(x)\,dx = \frac{7}{8}$

Answer: (a) $\mathbf{k = \frac{3}{64}}$ **(b) 3 (c)** $\frac{7}{8}$

6 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 1$, $y = -3$ so $a = 1$, $b = -3$

Through $(3, -4)$: $-4 = \frac{k}{2} - 3$

$k = (-4 + 3)(2) = -2$

$x = 0$: $y = -1$; $y = 0$: $x = \frac{1}{3}$

Answer: $k = -2$, $a = 1$, $b = -3$; $(0, -1)$ and $(\frac{1}{3}, 0)$

7 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 1$.
Find the gradients of the two possible tangents and the points of contact.

Tangent at $x = a$: $y - (a^2 + 1) = 2a(x - a)$. Through $(0, 0)$: $-a^2 - 1 = -2a^2$

So $a^2 = 1$ and $a = \pm 1$

Gradient $2a = \pm 2$

Answer: gradients ± 2 at points $(\pm 1, 2)$

8 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 19.6 m from the start, after 2 s and again after 5 s. Find (a) the speed of projection, (b) α to 1 d.p. Take $g = 9.8$.

Deceleration $a = g \sin \alpha$ (unknown). $s = ut - \frac{1}{2}at^2$ at both times:

$u \times 2 - \frac{1}{2}a \times 2^2 = u \times 5 - \frac{1}{2}a \times 5^2 = 19.6$

Subtracting: $u(5 - 2) = \frac{1}{2}a(25 - 4)$, so $u = \frac{1}{2}a(7)$

Substituting: $\frac{1}{2}a(7)(2) - \frac{1}{2}a \times 2^2 = 19.6$, so $\frac{1}{2}a \times 10 = 19.6$, $a = 3.92$

(a) $u = 1.96 \times 7 = 13.72$

(b) $9.8 \sin \alpha = 3.92$, $\sin \alpha = 0.4$, $\alpha = 23.6°$

Answer: (a) 13.72 m s^{-1} (b) 23.6°

9 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a minimum point at $(5, 1)$.
Find the coordinates of the corresponding point on:
(a) $y = f(-x)$
(b) $y = -f(x)$
(c) $y = f(x + 3)$

(a) $y = f(-x)$: $(-5, 1)$

(b) $y = -f(x)$: $(5, -1)$

(c) $y = f(x + 3)$: $(2, 1)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) $(-5, 1)$; (b) $(5, -1)$; (c) $(2, 1)$

10 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.1, independently. A box of 10 parts is rejected if it has at least 3 defective.
From 10 such boxes, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(10, 0.1)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.930 = 0.0702$

$W \sim B(10, 0.0702)$ is the number rejected

$P(W = 2) = C(10, 2)(0.0702)^2(1 - 0.0702)^8 = 0.124$

Answer: 0.124