Day 213 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(1 + 4x)^5$.

Term in x^2: ${}^5C_2 \times 1^3 \times (4x)^2$

$= 10 \times 1 \times 16$

Answer: 160

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + \sin x - 2 = 0$
for $0 \leq x \leq 2\pi$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - \sin x = 0$

Solving the quadratic: $\sin x = \frac{1}{2}$ or $\sin x = 0$

$\sin x = \frac{1}{2}$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = 0$: $x = 0, \pi, 2\pi$

Answer: $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$

3 Arithmetic modelling (All boards)

A charity raises £300 in its first year and in each later year raises £30 more than in the previous year.
(a) How much does it raise in year 16? (b) In which year does the total raised first exceed £10000?

(a) $u_{16} = 300 + 15 \times 30 = 750$

(b) $S_m = \frac{m}{2}(600 + (m - 1) \times 30) > 10000$

$S_{18} = 9990 < 10000$ and $S_{19} = 10830 > 10000$

Answer: (a) £750 (b) year 19

4 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a seed germinates is 0.15. In 16 independent trials, find the probability that this happens
(a) exactly 2 times (b) fewer than 2 times (c) at least 2 times.

Let $X \sim B(16, 0.15)$
(a) $P(X = 2) = 0.277$
(b) $P(X \leq 1) = 0.284$
(c) $1 - P(X \leq 1) = 0.716$
Answer: (a) 0.277 (b) 0.284 (c) 0.716

5 Differentiating sec, cosec, tan and cot (All boards)

$y = \cot(x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{4}$.

$\frac{dy}{dx} = -\text{cosec}^2(x)$

At $x = \frac{\pi}{4}$ the angle $x = \frac{\pi}{4}$, so the gradient is -2

Answer: $\frac{dy}{dx} = -\text{cosec}^2(x)$; -2

6 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 2 kg particle moves on a smooth horizontal line. At time t s a single force of $(36 - 6t)$ N acts along the line. When $t = 0$ its speed is 2 m s^{-1}. Find (a) its speed when $t = 10$, (b) its greatest speed.

$F = ma$: $a = (-6t + 36) \div 2$
$v = \int a\,dt = 18t - 1.5t^2 + c$; $v = 2$ when $t = 0$
(a) $v(10) = 32$
(b) Greatest speed when $a = 0$: $t = 6$
$v(6) = 56$
Answer: (a) 32 m s^{-1} (b) 56 m s^{-1}

7 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{4} u_n + 6$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{4} L + 6$ gives $L = \frac{6}{1 - \frac{1}{4}} = 8$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-8) \times \left(\frac{1}{4}\right)^{n-1}$

$|-8| \times \left(\frac{1}{4}\right)^{n-1} < \frac{1}{100}$

Check: n = 5 gives 0.03125, n = 6 gives 0.00781

Answer: (a) L = 8 (b) n = 6

8 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is $N(73, 3^2)$. An egg is classed as 'special' if its mass is less than 70.
11 eggs are chosen at random. Find the probability that exactly 2 are special (3 s.f.).

P(special) = P(X < 70) = P(Z < −1.00) = 0.159

Y ~ B(11, 0.159)

$P(Y = 2) = C(11, 2)(0.159)^2(0.841)^9 = 0.292$

Answer: 0.292

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

$n = \frac{12 - 9}{3 - 2} = 3$

At (2, 9): ln A = 9 − 3 × 2 = 3, so $A = e^3$

Answer: n = 3, A = e^3

10 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg carries a person of mass 75 kg. The cable tension is 4560 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 475 kg), taking upwards as positive: T − 475 g = 475 a
4560 − 4655 = 475 a, so a = −0.2

(a) a is negative, so the acceleration is 0.2 m s^{-2} downwards
(b) Person: R − 75 g = 75 a, so R = 75(9.8 − 0.2)

Answer: (a) 0.2 m s^{-2} downwards (b) 720 N